ISBN 978-0-7354-0810-4
ISSN 0094-243X

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TIM – 09

1262

AIP

PROCEEDINGS OF THE PHYSICS CONFERENCE

TIM – 09

Timisoara, Romania   27 – 28 November 2009

Universitatea de Vest din Timişoara

EDITORS

Madalin BUNOIU
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AIP
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All papers have been peer reviewed

SPONSORING ORGANIZATION
West University of Timisoara, Romania

American Institute of Physics

Melville, New York, 2010
AIP CONFERENCE PROCEEDINGS ■ VOLUME 1262

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L.C. Catalog Card No. 2010908302
ISBN 978-0-7354-0810-4
ISSN 0094-243X
Printed in the United States of America

CONTENTS

APPLIED PHYSICS

Preface

The Physics Conference TIM series has become very popular during the last years and is close to celebrate its first decade. The present proceedings volume contains selected papers presented at the Physics Conference TIM – 09 which is now one of the major events in sciences organized in the western region of Romania. The Conference was organized by the Faculty of Physics (www.physics.uvt.ro) from the West University of Timisoara (www.uvt.ro) between 27 of November to 28 of November 2009.

The aim of the Physics Conference TIM-09 (http://www.tim.uvt.ro) is to discuss actual national and international problems in physics, to bring together researchers and scientists from Romania and abroad, and to establish scientific contacts between them.

The scientific program of the conference included invited lectures, oral and poster presentations, as well as discussions on various topics of present interest, such as, but not limited to condensed matter physics and applications, theoretical and computational physics, and applied physics.

Invited lecturers, participants and collaborators to the contributions here presented were scientists from Bangladesh, Canada, France, Germany, Hungary, Ireland, Moldova, Romania, Russia, Serbia, Spain, Sweden, and United Kingdom.

The organizers would like to thank the keynote speakers and address their recognition to the Scientific Committee for the effort in reviewing the papers. A special note of thanks goes also to the Rector of the West University of Timisoara, Professor Ioan TALPOS and to the Dean of the Physics Faculty, Professor Dumitru VULCANOV for supporting the attempt to organize the conference in best conditions.

We would like also to address our appreciation to the authors of the papers because they contributed to the success of the conference.

Timisoara, May 19th, 2010

The Editors,

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NONCOMMUTATIVE GAUGE THEORY WITH COVARIANT STAR PRODUCT

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Abstract

We present a noncommutative gauge theory with covariant star product on a space-time with torsion. In order to obtain the covariant star product one imposes some restrictions on the connection of the space-time. Then, a noncommutative gauge theory is developed applying this product to the case of differential forms. Some comments on the advantages of using a space-time with torsion to describe the gravitational field are also given.

Keywords: noncommutative gauge theory, star product, differential forms

1. Introduction

The non-commutative gauge theory has been intensively studied in the last years with the hope that such a theory could offer the possibility to develop a quantum theory of gravity, or at least to give an idea of how this could be achieved [1 - 6]. There are two major candidates to quantum gravity: string theory [7] and loop quantum gravity [8]. Non-commutative geometry and in particular gauge theory of gravity are intimately connected with both these approaches and the overlaps are considerable [2]. String theory is one of the strongest motivations for considering non-commutative space-times geometries and non-commutative gravitation. The dynamics of the non-commutative gravity arising from string theory [10, 11] is much richer than some versions of the proposed non-commutative gravity. It is suspected that the reason for this is the non-covariance of the Moyal star product under space-time diffeomorphisms. A geometrical approach to non-commutative gravity, leading to a general theory of non-commutative Riemann surfaces, has been also proposed in [12] (for further developments, see [13, 14]).

One important problem is to develop a theory of gravity considering curved non-commutative space-times. The main difficulty is that non-commutative parameter $\theta^{\mu\nu}$ is usually taken to be constant, which breaks the Lorentz invariance of the commutation relations between coordinates

$$\left[x^{\mu}, x^{\nu}\right] = i\theta^{\mu\nu}, \tag{1.1}$$

and implicitly of any non-commutative field theory. One possible way to solve this problem is to consider $\theta^{\mu\nu}$ depending on coordinates, $\theta^{\mu\nu} = \theta^{\mu\nu}(x)$ and to use a covariant star product. In Ref. [15] such a product has been defined between differential forms and the property of associativity was verified up to the second order in $\theta^{\mu\nu}$.

In this paper we extend the result of Ref. [15] to case of Lie algebra valued differential forms following the same procedure as in our previous paper [16]. We will use a space-time with torsion only [9] which allows us to construct the teleparallel gravity [17-22], a much more convenient theory than general relativity to deal with the quantization problem [23].

2. Definition of the covariant star product

We consider a noncommutative space-time *M* endowed with the coordinates x^{μ}, $\mu = 0,1,2,3$ satisfying the commutation relation

$$\left[x^{\mu}, x^{\nu}\right] = i\theta^{\mu\nu}(x), \tag{2.1}$$

where $\theta^{\mu\nu}(x) = -\theta^{\nu\mu}(x)$ is a Poisson bivector [15]. The space-time is organized as a Poisson manifold by introducing the Poisson bracket between two functions $f(x)$ and $g(x)$ by

$$\{f, g\} = \theta^{\mu\nu}\partial_{\mu} f\,\partial_{\nu} g\,. \tag{2.2}$$

In order that the Poisson bracket satisfies the Jacobi identity, the bivector $\theta^{\mu\nu}(x)$ must obeys the condition [24-26]

$$\theta^{\mu\rho}\partial_{\rho}\theta^{\nu\sigma} + \theta^{\nu\rho}\partial_{\rho}\theta^{\sigma\mu} + \theta^{\sigma\rho}\partial_{\rho}\theta^{\mu\nu} = 0\,. \tag{2.3}$$

If a Poisson bracket is defined on *M*, then *M* is called a Poisson manifold (see [24] for mathematical details).

Let us suppose that $\Gamma^{\nu}_{\rho\sigma}$ define a non-symmetric connection on *M*. Then, we can construct the 1-forms of connection

$$\widetilde{\Gamma}^{\mu}_{\nu} = \Gamma^{\mu}_{\nu\rho} dx^{\rho}, \qquad \Gamma^{\mu}_{\nu} = dx^{\rho}\Gamma^{\mu}_{\rho\nu}\,, \tag{2.4}$$

and introduce two kinds of covariant derivatives $\widetilde{\nabla}$ and ∇, respectively. The curvatures for these two connections are

$$\widetilde{R}^{\nu}_{\lambda\rho\sigma} = \partial_{\rho}\Gamma^{\nu}_{\lambda\sigma} - \partial_{\sigma}\Gamma^{\nu}_{\lambda\rho} + \Gamma^{\nu}_{\tau\rho}\Gamma^{\tau}_{\lambda\sigma} - \Gamma^{\nu}_{\tau\sigma}\Gamma^{\tau}_{\lambda\rho}\,, \tag{2.5}$$

$$R^{\nu}_{\lambda\rho\sigma} = \partial_{\rho}\Gamma^{\nu}_{\sigma\lambda} - \partial_{\sigma}\Gamma^{\nu}_{\rho\lambda} + \Gamma^{\nu}_{\rho\tau}\Gamma^{\tau}_{\sigma\lambda} - \Gamma^{\nu}_{\sigma\tau}\Gamma^{\tau}_{\rho\lambda}\,. \tag{2.6}$$

The connection ∇ satisfies the identity [15]

$$\left[\nabla_{\mu}, \nabla_{\nu}\right]\alpha = -R^{\sigma}_{\rho\mu\nu} dx^{\rho} \wedge i_{\sigma}\alpha - T^{\rho}_{\mu\nu}\nabla_{\rho}\alpha\,, \tag{2.7}$$

and an analogous formula applies for $\widetilde{\nabla}$. Here, $T^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu}$ is the torsion and $i_{\sigma}\alpha$ denotes the interior product which maps the *k*-form α into a $(k-1)$-form [15, 16]. It has been proven that in order the Poisson bracket satisfies the Leibniz rule, the bi-vector $\theta^{\mu\nu}(x)$ must obeys the property

$$\widetilde{\nabla}_{\rho}\theta^{\mu\nu} = \partial_{\rho}\theta^{\mu\nu} + \Gamma^{\mu}_{\sigma\rho}\theta^{\sigma\nu} + \Gamma^{\nu}_{\sigma\rho}\theta^{\mu\sigma} \equiv 0\,, \tag{2.8}$$

i.e. $\theta^{\mu\nu}$ is covariant constant under $\widetilde{\nabla}$.

If α and β are two differential forms, then their Poisson bracket is defined as [15]

$$\{\alpha, \beta\} = \theta^{\mu\nu}\nabla_{\mu}\alpha \wedge \nabla_{\nu}\beta + (-1)^{|\alpha|}\widetilde{R}^{\mu\nu} \wedge (i_{\mu}\alpha) \wedge (i_{\nu}\beta)\,, \tag{2.9}$$

where $|\alpha|$ is the degree of the differential form α, and

$$\widetilde{R}^{\mu\nu} = \frac{1}{2}\widetilde{R}^{\mu\nu}_{\rho\sigma} dx^{\rho} \wedge dx^{\sigma}, \qquad \widetilde{R}^{\mu\nu}_{\rho\sigma} = \theta^{\mu\lambda}\widetilde{R}^{\nu}_{\lambda\rho\sigma}.$$

In order that (2.9) satisfies the graded Jacobi identity, the connection $\Gamma^{\rho}_{\mu\nu}$ must obey the following additional conditions [15]

$$R^{\nu}_{\lambda\rho\sigma} = 0, \tag{2.10}$$

$$\nabla_{\lambda}\widetilde{R}^{\mu\nu}_{\rho\sigma} = 0. \tag{2.11}$$

The covariant star product between arbitrary differential forms has the general form [15]

$$\alpha * \beta = \alpha \wedge \beta + \sum_{n=1}^{\infty} \hbar^n C_n(\alpha, \beta), \tag{2.12}$$

where $C_n(\alpha,\beta)$ are bilinear differential operators satisfying the generalized Moyal symmetry

$$C_n(\alpha,\beta) = (-1)^{|\alpha||\beta|+n} C_n(\beta,\alpha). \tag{2.13}$$

In addition, these operators must be chosen so that they satisfy the property of associativity

$$(\alpha * \beta) * \gamma = \alpha * (\beta * \gamma). \tag{2.14}$$

In this paper we consider the case when the symplectic manifold M has only torsion. Since the curvature $R^{\sigma}_{\mu\nu\rho}$ is vanishing [see Eq. (2.10)], one obtains the following relation between the curvature $\widetilde{R}$ and the torsion T [15]

$$\widetilde{R}^{\sigma}_{\mu\nu\rho} = \nabla_{\mu} T^{\sigma}_{\nu\rho}. \tag{2.15}$$

This relation shows that the curvature $\widetilde{R}^{\sigma}_{\mu\nu\rho}$ vanishes too if the torsion $T^{\sigma}_{\nu\rho}$ is covariant constant, i.e.

$$\nabla_{\mu} T^{\sigma}_{\nu\rho} = 0. \tag{2.16}$$

Therefore, if the torsion is covariant constant, the symplectic manifold M has only torsion but not curvature. In this case the bilinear differential operators $C_1(\alpha,\beta)$ and $C_2(\alpha,\beta)$ in the star product (2.12) have the expressions

$$C_1(\alpha,\beta) \equiv \{\alpha,\beta\} = \theta^{\mu\nu}\nabla_{\mu}\alpha \wedge \nabla_{\nu}\beta, \tag{2.17}$$

$$\begin{aligned} C_2(\alpha^a,\beta^b) &= \frac{1}{2}\theta^{\mu\nu}\theta^{\rho\sigma}\nabla_{\mu}\nabla_{\rho}\alpha \wedge \nabla_{\nu}\nabla_{\sigma}\beta + \\ &+ \frac{1}{3}\left(\theta^{\nu\rho}\nabla_{\rho}\theta^{\mu\sigma}\,\frac{1}{2}\theta^{\mu\rho}\theta^{\sigma\lambda}T^{\nu}_{\rho\lambda}\right)\left(\nabla_{\mu}\nabla_{\nu}\alpha \wedge \nabla_{\sigma}\beta - \nabla_{\nu}\alpha \wedge \nabla_{\mu}\nabla_{\sigma}\beta\right) \end{aligned} \tag{2.18}$$

In the following section we will develop an internal gauge theory for the case when the manifold M has only torsion but no torsion.

3. Noncommutative gauge theory

Suppose that we have an internal gauge group G whose infinitesimal generators T_a satisfy the algebra

$$[T_a, T_b] = i f^{c}_{ab} T_c, \quad a,b,c = 1,2,\ldots.,m, \tag{3.1}$$

with the structure constants $f^{a}_{bc} = -f^{a}_{cb}$, and that the Lie algebra valued infinitesimal parameter is

$$\hat{\lambda} = \hat{\lambda}^a T_a. \tag{3.2}$$

We use the hat symbol "^" to denote the non-commutative quantities of our gauge theory. The parameter $\hat{\lambda}$ is a 0-form, i.e. $\hat{\lambda}^a$ are functions of the coordinates x^{μ} on the symplectic manifold M.

The gauge transformation of the non-commutative Lie valued gauge potential $\hat{A} = \hat{A}_\mu^a(x) dx^\mu T_a = A^a T_a$ is given by

$$\hat{\delta}\hat{A} = d\hat{\lambda} - i\left[\hat{A}, \hat{\lambda}\right]_*, \tag{3.3}$$

where $[\ ,\ ]_*$ is the commutator defined with the covariant star product (2.12). The covariant derivative of the 1-form gauge potentials is

$$\nabla_\mu \hat{A}^a = \left(\partial_\mu \hat{A}_\nu^a - \Gamma_{\mu\nu}^\rho \hat{A}_\rho^a\right) dx^\nu \equiv \left(\nabla_\mu \hat{A}_\nu^a\right) dx^\nu . \tag{3.4}$$

We define then the curvature 2-form $\hat{F}$ of the gauge potentials by

$$\hat{F} = \frac{1}{2}\hat{F}_{\mu\nu}\, dx^\mu \wedge dx^\nu = d\hat{A} - \frac{i}{2}\left[\hat{A}, \hat{A}\right]_*. \tag{3.5}$$

whose transformation law is

$$\hat{\delta}\hat{F} = i\left[\hat{\lambda}, \hat{F}\right]_*. \tag{3.6}$$

We can verify that $\hat{F}$ satisfies the deformed Bianchi identity [16]

$$d\hat{F} - i\left[\hat{A}, \hat{F}\right]_* = 0 . \tag{3.7}$$

We also remark that in zeroth order we obtain from (3.7) the usual Bianchi identity.

If $\hat{G}^{\nu\rho}$ is a noncommutative gauge covariant metric on M, then the Yang-Mills invariant action of the gauge fields $A_\mu^a(x)$ can be chosen as

$$\hat{S}_{NC} = -\frac{1}{2g^2} Tr \int d^4x \left(\hat{G} * \hat{F} * \hat{G} * \hat{F}\right) = -\frac{1}{4g^2} \int d^4x \left(\int d^4x \left(\hat{G}^{\mu\rho} * \hat{F}_{\rho\nu} * \hat{G}^{\nu\sigma} * \hat{F}_{\sigma\mu}\right)\right), \tag{5.4}$$

where g is the gauge coupling constant, and we used the normalization property $Tr(T_a T_b) = \frac{1}{2}\delta_{ab} I$. Imposing then the variational principle $\hat{\delta}_{\hat{A}} \hat{S}_{NC} = 0$ with respect to the non-commutative gauge fields $\hat{A}_\mu^a$, we can obtain the non-commutative Yang-Mills field equations. Other models for a noncommutative gauge theory of gravity are given in [30-35].

4. **Illustrative example**

Suppose that on the manifold M we have defined the gauge fields $e_\mu^a, \omega_\mu^{ab} = -\omega_\mu^{ba}$ [27-29] and fix the gauge $\omega_\mu^{ab} = 0$ [9]. We define the connection coefficients

$$\Gamma_{\mu\nu}^\rho = \bar{e}_a^\rho \partial_\nu e_\mu^a , \tag{4.1}$$

where $\bar{e}_a^\rho$ denotes the inverse of e_μ^a. Then, we consider the case of spherically symmetry and choose the gauge fields e_μ^a as

$$e_\mu^a = diag\left[A, 1, 1, \frac{1}{A}\right], \quad \bar{e}_a^\mu = diag\left[\frac{1}{A}, 1, 1, A\right], \tag{4.3}$$

where $A = A(r)$ is a function depending only on the radial coordinate r. The non-null components of the connection coefficients and the torsion are respectively

$$\Gamma_{10}^0 = -\frac{A'}{A}, \quad \Gamma_{11}^1 = \frac{A'}{A}, \quad T_{01}^0 = -T_{10}^0 = \frac{A'}{A} \tag{4.4}$$

Also, the non-null components of the two curvatures are

$$\widetilde{R}_{101}^0 = -\widetilde{R}_{110}^0 = \frac{AA'' - 2A'^2}{A^2}, \quad R_{\mu\nu\rho}^\lambda = 0 , \tag{4.5}$$

where we denoted the first and second derivatives of $A(r)$ by A' and A'' respectively.

Suppose now that only non-null parameters are $\theta^{10} = -\theta^{01} = \dfrac{1}{A}$; then we have

$$\widetilde{\nabla}_1 \theta^{01} = -\widetilde{\nabla}_1 \theta^{10} = 0, \quad \nabla_1 \theta^{01} = -\nabla_1 \theta^{10} = -\frac{A'}{A^2}, \tag{4.6}$$

If we use (4.5), w can see that the tensor $\widetilde{R}^{\mu\nu}_{\rho\sigma} = \theta^{\mu\lambda} \widetilde{R}^{\nu}_{\lambda\rho\sigma}$ is vanishing, and implicitly $\nabla_\lambda \widetilde{R}^{\mu\nu}_{\rho\sigma} = 0$, if

$$AA'' - 2A'^2 = 0. \tag{4.7}$$

The solutions of this equation is

$$A(r) = -\frac{1}{ar+b}, \tag{4.8}$$

where a and b are two arbitrary constants of integration. Therefore, in our example, the conditions necessary to define a covariant star product on a symplectic manifold M completely determine its connection. In addition, it is very interesting to see that the covariant derivative of the torsion has the following non-null components

$$\nabla_1 T^0_{10} = -\nabla_1 T^0_{01} = \frac{AA'' - 2A'^2}{A^2}. \tag{4.9}$$

Then, tacking into account the equation (4.7), we conclude that the torsion is covariant constant, $\nabla_\mu T^\nu_{\rho\sigma} = 0$, a result which is in concordance with the condition (2.16).

All the calculations in this section have been performed by using an analytical program conceived for GRTensor II package of the Maple platform. Specific routines have been written and adapted for Maple.

Acknowledgements. The author acknowledges the support of CNCSIS-UEFISCSU Grant ID-620 of the Ministry of Education and Research of Romania.

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