Eur. Phys. J. C (2008) 57: 809–815
DOI 10.1140/epjc/s10052-008-0690-7

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Towards a quantization of gauge fields on de Sitter group by functional integral method

Viorel Chiritoiu[1,a], **Gheorghe Zet**[2,b]

[1]Technical Physics Department "Politehnica" University Timisoara, Timisoara, Romania
[2]Department of Physics "Gh. Asachi" Technical University Iasi, Iasi, Romania

Received: 17 April 2008 / Revised: 17 July 2008 / Published online: 3 September 2008

Abstract A formulation of the de Sitter symmetry as a purely inner symmetry defined on a fixed Minkowski space-time is presented. We define the generators of the de Sitter group and write the structure equations using a constant deformation parameter λ. The conserved gauge currents are calculated, and their physical meaning is given. Local gauge transformations and the corresponding covariant derivative depending on the gauge fields are also obtained. We study the behavior of gauge fields, the torsion and curvature tensors and give a regularization technique in terms of the ζ function.

1 Introduction

All known fundamental interactions, excepting gravitation, are mediated by gauge fields. Looking at the possible candidate groups for a gauge theory to describe gravitation, the Poincaré group is the obvious choice. This theory allows us to obtain Einstein's equations in a particular case, and Newton's law in the static non-relativistic limit. But this theory is not quantizable.

The standard procedure of adding new terms and obtaining a consistent and renormalizable theory can be applied if we consider one of the de Sitter groups, $SO(4,1)$ and $SO(3,2)$. Following this procedure, we shall use in this paper as gauge group of gravitation a deformation of the de Sitter group $SO(4,1)$, determined by a constant parameter λ.

Quantum field theories on the $dS(4,1)$—de Sitter and $dS(3,2)$—anti-de Sitter space-times originate from the paper of Dirac [1], who discussed the electron wave equation in de Sitter space. There is a difference between this approach and the gauge theory. The difference consists in the fact that the electron wave equation involves only the angular momentum operator, while the gauge theory includes both this operator and the ordinary momentum operator.

Also a method of quantization of fields using the stereographic projection was proposed. Such a formulation was developed for the first time by Adler [2] to study massless Euclidean QED (quantum electrodynamics) on a hypersphere in a 5-dimensional space. This idea was then carried out by other authors [3–6] in order to obtain a formulation of non-abelian gauge theories on both de Sitter and anti-de Sitter spaces. Effectively, the theory on the flat Minkowski space is projected onto the de Sitter space by a stereographic transformation. In [6] it is shown that the variables in the gauge sector (like potentials, field strengths, etc.) in the two descriptions are related by rules similar to the usual tensor analysis. The role of the metric is played in this formalism by conformal Killing vectors. Analogously, the quantities in the matter (fermionic) sector are related by conformal Killing spinors. It is stressed that the extension of this analysis to the quantum field theory is quite nontrivial.

It is also important to remark that the limit of the de Sitter curvature going to zero is equivalent to the limit in which the cosmological constant goes to zero. Therefore, in this limit, the de Sitter and anti-de Sitter groups reduce to the Poincaré group, and the de Sitter spaces reduce to Minkowski space.

Our work extends the analysis given in [7, 8] for Poincaré gauge theory in Minkowski space-time to the case of de Sitter gauge group using a stereographic projection. Because any quantization of a gauge theory requires the issue of gauge fixing, we are going to apply to our approach, in a forthcoming paper [17], the BRST gauge fixing procedure in a manner similar to that developed in [9].

This paper is organized as follows: in Sect. 2 the connection between stereographic projection and the generators of de Sitter group is presented and the commutation relations between the generators are obtained. We present the de Sitter symmetry as a pure inner symmetry in Sect. 3. We also

[a] e-mail: viorel.chiritoiu@et.upt.ro
[b] e-mail: gzet@phys.tuiasi.ro

calculate the conserved current, the angular momentum tensor and functions $\delta f^{\gamma}(x)$ which ensure the gauge invariance of the action.

In Sects. 4 and 5 we define the covariant derivative and the gauge fields, and we determine their local infinitesimal transformations. We also introduce the torsion tensor, the curvature tensor and the strength field tensor, quantities which become dependent of parameter λ. If the parameter λ is close to zero, all these results are similar with results obtained by using the Poincaré group [7] as a symmetry group.

We then write the invariant matter action for scalar, spinor and vector fields in Sect. 6. The occurrence of the new gauge field e can be seen in every type of action. A renormalization technique of gauge fields using the ζ function method is presented in Sect. 7.

2 The de Sitter spaces and groups

A de Sitter universe can be defined as a pseudosphere in a 5-dimensional flat space with Cartesian coordinates $\xi^A = (\xi^0, \xi^1, \xi^2, \xi^3, \xi^5)$. Denoting $\eta_{ab} = \mathrm{diag}(-1,1,1,1)$ with indices $a, b = 0,1,2,3$ and $\eta_{55} = \epsilon$, the coordinates ξ^A satisfy

$$\eta_{AB}\xi^A\xi^B = \eta_{ab}\xi^a\xi^b + \epsilon\big(\xi^5\big) = \epsilon\mathcal{R}^2, \tag{1}$$

where $\epsilon = 1$ for de Sitter space, $dS(4,1)$, and $\epsilon = -1$ for anti-de Sitter space, $dS(3,2)$. The de Sitter space, $dS(4,1)$, with the diagonal metric $\eta_{AB} = \mathrm{diag}(-1,1,1,1,1)$ has the pseudo-orthogonal group $SO(4,1)$ as the group of motion. The anti-de Sitter space $dS(3,2)$ has the diagonal metric $\eta_{AB} = \mathrm{diag}(-1,1,1,1,-1)$ and the group $SO(3,2)$ as the group of motion. The 4-dimensional stereographic coordinates x^μ are obtained by projecting a de Sitter surface into a Minkowski space. The Minkowski metric is $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ with indices $\mu, \nu = 0,1,2,3$. The most important connection equations between the two types of coordinates are

$$\xi^a = n(x)\delta^a{}_\mu x^\mu; \qquad \xi^5 = -\mathcal{R}n(x)\left(1 - \epsilon\frac{\sigma^2}{4\mathcal{R}^2}\right), \tag{2}$$

where

$$n(x) = \frac{1}{1+\epsilon\frac{\sigma^2}{4\mathcal{R}^2}}, \qquad \sigma^2 = \eta_{\mu\nu}x^\mu x^\nu. \tag{3}$$

Let us consider a gauge theory of gravitation which has the de Sitter group as a local symmetry group, with generators [4, 5]

$$J_{\alpha 5} \equiv \Pi_\alpha = p_\alpha + \lambda^2 K_\alpha + \lambda\Sigma_{\alpha 5} \tag{4}$$

and

$$J_{\alpha\beta} \equiv m_{\alpha\beta} = i(x_\alpha\partial_\beta - x_\beta\partial_\alpha) + \frac{1}{2}\Sigma_{\alpha\beta}, \tag{5}$$

where

$$p_\alpha = i\partial_\alpha; \qquad K_\alpha = t_\alpha{}^\gamma p_\gamma, \qquad t_\alpha{}^\gamma = \big(2\eta_{\alpha\beta}x^\beta x^\gamma - \sigma^2\delta_\alpha{}^\gamma\big). \tag{6}$$

Because the quantities $t_\alpha{}^\beta$ depend only on the coordinates, we have

$$\big[t_\alpha{}^\beta, t_\gamma{}^\delta\big] = 0. \tag{7}$$

In order to discuss the commutation relations between the de Sitter generators some useful relations are summarized here:

$$\begin{aligned}
&[p_\alpha, p_\beta] = 0,\\
&[K_\alpha, K_\beta] = 0,\\
&[p_\alpha, K_\beta] + [K_\alpha, p_\beta] = 4L_{\alpha\beta},\\
&[p_\alpha, L_{\beta\gamma}] = \eta_{\alpha\beta}p_\gamma - \eta_{\alpha\gamma}p_\beta,\\
&[K_\alpha, L_{\beta\gamma}] = \eta_{\alpha\beta}K_\gamma - \eta_{\alpha\gamma}K_\beta,\\
&[p_\alpha, \Sigma_{\beta\gamma}] = 0,\\
&[p_\alpha, \Sigma_{\beta 5}] = 0,\\
&[K_\alpha, \Sigma_{\beta\gamma}] = 0,\\
&[K_\alpha, \Sigma_{\beta 5}] = 0,\\
&[\Sigma_{\alpha 5}, \Sigma_{\beta 5}] = -2i\Sigma_{\alpha\beta},\\
&[\Sigma_{\alpha 5}, \Sigma_{\beta\gamma}] = 2i(\eta_{\alpha\beta}\Sigma_{\gamma 5} - \eta_{\alpha\gamma}\Sigma_{\beta 5}),
\end{aligned} \tag{8}$$

where $L_{\alpha\beta} = i(x_\alpha\partial_\beta - x_\beta\partial_\alpha)$. Using (8), the commutation relations between de Sitter generators become [4, 8]

$$\begin{aligned}
&[\Pi_\alpha, \Pi_\beta] = -4i\lambda^2 m_{\alpha\beta},\\
&[\Pi_\alpha, m_{\beta\gamma}] = i(\eta_{\alpha\beta}\Pi_\gamma - \eta_{\alpha\gamma}\Pi_\beta),\\
&[m_{\alpha\beta}, m_{\gamma\delta}] = i(\eta_{\beta\gamma}m_{\alpha\delta} - \eta_{\beta\delta}m_{\alpha\gamma}\\
&\qquad + \eta_{\alpha\gamma}m_{\delta\beta} - \eta_{\alpha\delta}m_{\gamma\beta}),\\
&[\Sigma_{\alpha\beta}, \Sigma_{\gamma\delta}] = 2i(\eta_{\beta\gamma}\Sigma_{\alpha\delta} - \eta_{\beta\delta}\Sigma_{\alpha\gamma}\\
&\qquad + \eta_{\alpha\gamma}\Sigma_{\delta\beta} - \eta_{\alpha\delta}\Sigma_{\gamma\beta}).
\end{aligned} \tag{9}$$

In the limit $\lambda \to 0$, the process named contraction, the de Sitter algebra is transformed into the Poincaré algebra and $\Pi_\alpha = p_\alpha$ [4, 8].

3 Pure inner de Sitter symmetry

Considering a set of fields φ_j, with $j = 1, \ldots, n$, their dynamics will be specified by the action

$$S_{\mathrm{M}} = \int \mathrm{d}^4x\,\mathcal{L}_{\mathrm{M}}(x, \varphi_j, \partial_\alpha\varphi_j). \tag{10}$$

Then $\delta S_{\mathrm{M}}=0$ yields the equations of motion. If there are functions $\delta f^{\gamma}(x)$ for which

$$\begin{aligned} &\mathrm{d}^4x'\mathcal{L}_{\mathrm{M}}\big(x',\varphi'_j(x'),\partial'_\alpha\varphi'_j(x')\big)\\ &\quad=\mathrm{d}^4x\big[\mathcal{L}_{\mathrm{M}}\big(x,\varphi_j(x),\partial_\alpha\varphi_j(x)\big)+\partial_\gamma\delta f^{\gamma}(x)\big] \end{aligned} \tag{11}$$

holds, then the variation δS_{M} of the action can be written as [12, 13]

$$\begin{aligned} \delta S_{\mathrm{M}}=&-\int\mathrm{d}^4x\,\partial_\gamma\left(-\frac{\partial\mathcal{L}}{\partial(\partial_\gamma\varphi_j)}\cdot\delta_T\varphi_j\right)\\ &-\int\mathrm{d}^4x\,\partial_\gamma\left(\frac{\partial\mathcal{L}}{\partial(\partial_\gamma\varphi_j)}\cdot\partial_\alpha\varphi_j-\eta^{\gamma}{}_{\alpha}\mathcal{L}\right)\delta x^{\alpha}\\ &+\int\mathrm{d}^4x\,\partial_\gamma\delta f^{\gamma}. \end{aligned} \tag{12}$$

From the Noether theorem [10, 11], we obtain the associated conserved current J^{γ}:

$$J^{\gamma}=-\frac{\partial\mathcal{L}_{\mathrm{M}}}{\partial(\partial_\gamma\varphi_j)}\cdot\delta_T\varphi_j+\delta f^{\gamma}+\Theta^{\gamma}{}_{\alpha}\cdot\delta x^{\alpha}, \tag{13}$$

where $\Theta^{\gamma}{}_{\alpha}$ is the energy-momentum tensor

$$\Theta^{\gamma}{}_{\alpha}=\frac{\partial\mathcal{L}_{\mathrm{M}}}{\partial(\partial_\gamma\varphi_j)}\cdot\partial_\alpha\varphi_j-\eta^{\gamma}{}_{\alpha}\cdot\mathcal{L}_{\mathrm{M}}. \tag{14}$$

The global de Sitter transformations are given by the relation

$$\begin{aligned} &x^{\alpha}\to x'_{\alpha}=x^{\alpha}+\varepsilon^{\alpha}+\lambda^2\varepsilon^{\xi}t_{\xi}{}^{\alpha}+\omega^{\alpha}{}_{\beta}x^{\beta},\\ &\varphi_j(x)\to\varphi'_j(x')=\varphi_j(x)-\frac{i}{4}\omega^{\alpha\beta}\Sigma_{\alpha\beta}\varphi_j(x)\\ &\qquad\qquad+i\lambda\varepsilon^{\alpha}\Sigma_{\alpha5}\varphi_j(x). \end{aligned} \tag{15}$$

In this case the coordinates x^{α} and the fields $\varphi_j(x)$ transform as

$$\delta x^{\alpha}=\varepsilon^{\alpha}+\lambda^2\varepsilon^{\xi}t_{\xi}{}^{\alpha}+\omega^{\alpha}{}_{\beta}x^{\beta} \tag{16}$$

and

$$\delta_T\varphi_j=-\frac{i}{4}\omega^{\alpha\beta}\Sigma_{\alpha\beta}\varphi_j(x)+i\lambda\varepsilon^{\alpha}\Sigma_{\alpha5}\varphi_j(x), \tag{17}$$

respectively. It can be verified that the invariance of the action under these global de Sitter transformations is ensured if

$$\delta f^{\gamma}=0. \tag{18}$$

On the other hand, if the de Sitter gauge symmetry [14, 15] is considered as a pure inner symmetry, then the infinitesimal transformations are [7]

$$\begin{aligned} &x^{\alpha}\to x'_{\alpha}=x^{\alpha},\\ &\varphi_j(x)\to\varphi'_j(x)=\varphi_j(x)\\ &\qquad-\big(\varepsilon^{\alpha}+\lambda^2\varepsilon^{\xi}t_{\xi}{}^{\alpha}+\omega^{\alpha}{}_{\beta}x^{\beta}\big)\cdot\partial_\alpha\varphi_j(x)\\ &\qquad-\frac{i}{4}\omega^{\alpha\beta}\Sigma_{\alpha\beta}\varphi_j(x)+i\lambda\varepsilon^{\alpha}\Sigma_{\alpha5}\varphi_j(x), \end{aligned} \tag{19}$$

where the parameters of the de Sitter group ε^{α} and $\omega^{\alpha\beta}$ now depend on x. In this case, the variation of the action is

$$\begin{aligned} \delta S_{\mathrm{M}}=&-\int\mathrm{d}^4x\big(\varepsilon^{\alpha}+\lambda^2\varepsilon^{\xi}t_{\xi}{}^{\alpha}+\omega^{\alpha}{}_{\beta}x^{\beta}\big)\delta^{\gamma}{}_{\alpha}\partial_\gamma\mathcal{L}\\ &-\lambda^2\varepsilon^{\xi}\int\mathrm{d}^4x\,\mathcal{L}\delta^{\gamma}{}_{\alpha}\partial_\gamma t_{\xi}{}^{\alpha}+\int\mathrm{d}^4x\,\partial_\gamma\delta f^{\gamma}(x), \end{aligned} \tag{20}$$

and the invariance of the action, $\delta S_{\mathrm{M}}=0$, requires

$$\delta f^{\gamma}=-\delta^{\gamma}{}_{\alpha}\big(\varepsilon^{\alpha}+\lambda^2\varepsilon^{\xi}t_{\xi}{}^{\alpha}+\omega^{\alpha}{}_{\beta}x^{\beta}\big)\mathcal{L}. \tag{21}$$

Then, the conserved current J^{γ} can be written as

$$\begin{aligned} J^{\gamma}=&\,\Theta^{\gamma}{}_{\alpha}\cdot\varepsilon^{\alpha}+\lambda^2\Theta^{\gamma}{}_{\alpha}\cdot\varepsilon^{\xi}\cdot t_{\xi}{}^{\alpha}+\frac{1}{2}\mathcal{M}^{\gamma}{}_{\alpha\beta}\cdot\omega^{\alpha\beta}\\ &-i\lambda\varepsilon^{\alpha}\frac{\partial\mathcal{L}}{\partial(\partial_\gamma\varphi_j)}\Sigma_{\alpha5}\varphi_j, \end{aligned} \tag{22}$$

where the angular momentum tensor is

$$\mathcal{M}^{\gamma}{}_{\alpha\beta}=\Theta^{\gamma}{}_{\alpha}x_{\beta}-\Theta^{\gamma}{}_{\beta}x_{\alpha}+\frac{i}{2}\frac{\partial\mathcal{L}_{\mathrm{M}}}{\partial(\partial_\gamma\varphi_j)}\Sigma_{\alpha\beta}\varphi_j. \tag{23}$$

We can rewrite the gauge transformations (3) as

$$\begin{aligned} &x^{\alpha}\to x'_{\alpha}=x^{\alpha},\\ &\varphi_j(x)\to\varphi'_j=\big((1+\Theta)\varphi_j\big)(x), \end{aligned} \tag{24}$$

with

$$\begin{aligned} \Theta(x)=&-\big\{\varepsilon^{\gamma}(x)+\lambda^2\varepsilon^{\xi}(x)t_{\xi}{}^{\gamma}+\omega^{\gamma\delta}(x)x_{\delta}\big\}\cdot\partial_\gamma\\ &-\frac{i}{4}\omega^{\gamma\delta}(x)\Sigma_{\gamma\delta}+i\lambda\varepsilon^{\gamma}(x)\Sigma_{\gamma5}\\ =&\,i\varepsilon^{\gamma}(x)\cdot\Pi_{\gamma}-\frac{i}{2}\omega^{\gamma\delta}(x)\cdot m_{\gamma\delta}. \end{aligned} \tag{25}$$

In this case

$$\begin{aligned} &\mathrm{d}^4x\mathcal{L}_{\mathrm{M}}\big(\varphi'_j(x),\partial_\alpha\varphi'_j(x)\big)\\ &\quad=\mathrm{d}^4x\mathcal{L}_{\mathrm{M}}\big(\varphi_j(x),\partial_\alpha\varphi_j(x)\big)\\ &\qquad-\mathrm{d}^4x\big(\varepsilon^{\gamma}+\lambda^2\varepsilon^{\xi}t_{\xi}{}^{\gamma}+\omega^{\gamma}{}_{\beta}x^{\beta}\big)\partial_\gamma\mathcal{L}_{\mathrm{M}}\big(\varphi_j(x),\partial_\alpha\varphi_j(x)\big)\\ &\qquad-\lambda^2\,\mathrm{d}^4x\,\varepsilon^{\xi}\mathcal{L}_{\mathrm{M}}\big(\varphi_j(x),\partial_\alpha\varphi_j(x)\big)\partial_\gamma t_{\xi}{}^{\gamma}(x). \end{aligned} \tag{26}$$

Therefore, (26) does not lead to invariance of the action: the second term on the right hand side is no longer a pure divergence.

4 Local de Sitter gauge invariance. The covariant derivative $\tilde{\nabla}_\alpha$ and its decomposition as a function of Π_α and $m_{\gamma\delta}$

As $\Theta(x)$ can be written as a function of de Sitter generators, we define the covariant derivative $\tilde{\nabla}_\alpha$ [7, 12]

$$\tilde{\nabla}_\alpha = \partial_\alpha + B_\alpha, \tag{27}$$

together with the decomposition of B_α as a function of Π_α and $m_{\gamma\delta}$:

$$B_\alpha \equiv -i B_\alpha{}^\gamma \cdot \Pi_\gamma + \frac{i}{2} B_\alpha{}^{\gamma\delta} \cdot m_{\gamma\delta}. \tag{28}$$

Therefore, we introduced the 16 fields $B_\alpha{}^\gamma$ for local translations and the 24 antisymmetric fields $B_\alpha{}^{\gamma\delta}$ for the local Lorentz rotations. We will determine now the behavior of B_α under local gauge transformations. Using (25), we can write

$$\begin{aligned} \delta B_\alpha &= [\Theta, \partial_\alpha + B_\alpha] \\ &= \Theta\partial_\alpha - \partial_\alpha\Theta + [\Theta, B_\alpha]. \end{aligned} \tag{29}$$

Then we easily find that

$$\begin{aligned} \Theta\partial_\alpha &= i\varepsilon^\gamma\big(i\partial_\gamma + i\lambda^2 t_\gamma{}^\delta\partial_\delta + \lambda\Sigma_{\gamma5}\big)\partial_\alpha + \omega_\alpha{}^\beta\partial_\beta \\ &= \omega_\alpha{}^\beta\partial_\beta, \end{aligned} \tag{30}$$

and therefore

$$\begin{aligned} \partial_\alpha\Theta &= i\partial_\alpha\varepsilon^\gamma \cdot \Pi_\gamma + i\lambda^2\varepsilon^\xi \cdot \partial_\alpha t_\xi{}^\gamma \cdot p_\gamma - \frac{i}{2}\partial_\alpha\omega^{\gamma\delta} \cdot m_{\gamma\delta} \\ &\quad + \omega_\alpha{}^\beta\partial_\beta. \end{aligned} \tag{31}$$

After evaluation of the commutator

$$\begin{aligned} [\Theta, B_\alpha] = \bigg[& i\varepsilon^\gamma \cdot \Pi_\gamma - \frac{i}{2}\omega^{\gamma\delta} \cdot m_{\gamma\delta}, \\ & - i B_\alpha{}^\varepsilon \cdot \Pi_\varepsilon + \frac{i}{2} B_\alpha{}^{\varepsilon\xi} \cdot m_{\varepsilon\xi} \bigg], \end{aligned} \tag{32}$$

and writing the variation of the gauge fields as a function of the generators Π_γ and $m_{\gamma\delta}$,

$$\delta B_\alpha \equiv -i\delta B_\alpha{}^\gamma \cdot \Pi_\gamma + \frac{i}{2}\delta B_\alpha{}^{\gamma\delta} \cdot m_{\gamma\delta}, \tag{33}$$

the transformation laws of the gauge fields are given by

$$\begin{aligned} \delta B_\alpha{}^\gamma = {} & \partial_\alpha\varepsilon^\gamma + \lambda^2\varepsilon^\xi \cdot \partial_\alpha t_\xi{}^\gamma - \varepsilon^\xi \cdot \partial_\xi B_\alpha{}^\gamma \\ & - \lambda^2\varepsilon^\xi t_\xi{}^\delta \cdot \partial_\delta B_\alpha{}^\gamma - \lambda^2\varepsilon^\varepsilon B_\alpha{}^\xi \cdot \partial_\varepsilon t_\xi{}^\gamma \\ & + B_\alpha{}^\xi \cdot \partial_\xi\varepsilon^\gamma + \lambda^2 B_\alpha{}^\varepsilon t_\varepsilon{}^\xi \cdot \partial_\xi\varepsilon^\gamma \\ & + \lambda^2 B_\alpha{}^\varepsilon\varepsilon^\xi \cdot \partial_\varepsilon t_\xi{}^\gamma - B_\alpha{}^{\varepsilon\xi}x_\varepsilon \cdot \partial_\xi\varepsilon^\gamma - \varepsilon_\delta B_\alpha{}^{\gamma\delta} \\ & - \lambda^2\varepsilon^\delta B_\alpha{}^{\varepsilon\xi}x_\varepsilon \cdot \partial_\xi t_\delta{}^\gamma + \omega^{\varepsilon\xi}x_\varepsilon\partial_\xi B_\alpha{}^\gamma + \omega_\alpha{}^\beta B_\beta{}^\gamma \\ & + \omega_\xi{}^\gamma B_\alpha{}^\xi + \lambda^2 B_\alpha{}^\delta\omega^{\varepsilon\xi}x_\varepsilon \cdot \partial_\xi t_\delta{}^\gamma, \end{aligned} \tag{34}$$

and

$$\begin{aligned} \delta B_\alpha{}^{\gamma\delta} = {} & \partial_\alpha\omega^{\gamma\delta} - \varepsilon^\xi \cdot \partial_\xi B_\alpha{}^{\gamma\delta} - \lambda^2\varepsilon^\varepsilon t_\varepsilon{}^\xi \cdot \partial_\xi B_\alpha{}^{\gamma\delta} \\ & + B_\alpha{}^\xi \cdot \partial_\xi\omega^{\gamma\delta} + \lambda^2 B_\alpha{}^\varepsilon t_\varepsilon{}^\xi \cdot \partial_\xi\omega^{\gamma\delta} \\ & + \omega^{\varepsilon\xi}x_\varepsilon \cdot \partial_\xi B_\alpha{}^{\gamma\delta} - B_\alpha{}^{\varepsilon\xi}x_\varepsilon \cdot \partial_\xi\omega^{\gamma\delta} \\ & + \omega^\gamma{}_\xi B_\alpha{}^{\xi\delta} + \omega^\delta{}_\xi B_\alpha{}^{\gamma\xi} + \omega_\alpha{}^\beta B_\beta{}^{\gamma\delta}. \end{aligned} \tag{35}$$

5 Local de Sitter gauge invariance. The covariant derivative $\tilde{\nabla}_\alpha$ and its decomposition as a function of ∂_α, $\Sigma_{\gamma\delta}$ and $\Sigma_{t\gamma5}$

In this section we first decompose the covariant derivative $\tilde{\nabla}_\alpha$ in terms of ∂_α, $\Sigma_{\gamma\delta}$ and $\Sigma_{\gamma5}$, by introducing an extra gauge field $e_\alpha{}^\gamma$. We recast $\tilde{\nabla}_\alpha$ in the form [7, 8]

$$\tilde{\nabla}_\alpha = e_\alpha{}^\gamma\partial_\gamma + \frac{i}{4}B_\alpha{}^{\gamma\delta}\Sigma_{\gamma\delta} - i\lambda B_\alpha{}^\gamma\Sigma_{\gamma5}, \tag{36}$$

where

$$e_\alpha{}^\gamma = \delta_\alpha{}^\gamma + B_\alpha{}^\gamma + \lambda^2 B_\alpha{}^\beta t_\beta{}^\gamma + B_\alpha{}^{\gamma\delta}x_\delta. \tag{37}$$

Abbreviating

$$\begin{aligned} & d_\alpha \equiv e_\alpha{}^\gamma\partial_\gamma, \qquad B_\alpha \equiv \frac{i}{4}B_\alpha{}^{\gamma\delta}\Sigma_{\gamma\delta}, \\ & C_\alpha \equiv -i\lambda B_\alpha{}^\gamma\Sigma_{\gamma5}, \end{aligned} \tag{38}$$

we can write the covariant derivative $\tilde{\nabla}_\alpha$ in a simpler form:

$$\tilde{\nabla}_\alpha = d_\alpha + B_\alpha + C_\alpha. \tag{39}$$

The variation of $e_\alpha{}^\gamma$ becomes

$$\begin{aligned} \delta e_\alpha{}^\gamma = {} & e_\alpha{}^\xi \cdot \partial_\xi\big(\varepsilon^\gamma + \lambda^2\varepsilon^\varepsilon t_\varepsilon{}^\gamma + \omega^{\gamma\delta}x_\delta\big) \\ & - \big(\varepsilon^\xi + \lambda^2\varepsilon^\varepsilon t_\varepsilon{}^\xi + \omega^{\xi\eta}x_\eta\big) \cdot \partial_\xi e_\alpha{}^\gamma + \omega_\alpha{}^\xi e_\xi{}^\gamma; \end{aligned} \tag{40}$$

it is expressed in terms of $e_\alpha{}^\gamma$ only. For the variation of $B_\alpha{}^{\gamma\delta}$ we obtain

$$\begin{aligned} \delta B_\alpha{}^{\gamma\delta} = {} & e_\alpha{}^\xi\partial_\xi\omega^{\gamma\delta} - \big(\varepsilon^\xi + \lambda^2\varepsilon^\varepsilon t_\varepsilon{}^\xi + \omega^{\xi\eta}x_\eta\big)\partial_\xi B_\alpha{}^{\gamma\delta} \\ & + \omega_\alpha{}^\xi B_\xi{}^{\gamma\delta} + \omega^\gamma{}_\xi B_\alpha{}^{\xi\delta} + \omega^\delta{}_\xi B_\alpha{}^{\gamma\xi}. \end{aligned} \tag{41}$$

Because the determinant $\det e^{-1}$ will enter into the locally de Sitter invariant action, we give its transformation behavior here:

$$\delta \det e^{-1} = -\det e^{-1} \cdot \partial_\xi \big(\varepsilon^\xi + \lambda^2 \varepsilon^\varepsilon t_\varepsilon{}^\xi + \omega^{\xi\eta} x_\eta\big) - \big(\varepsilon^\xi + \lambda^2 \varepsilon^\varepsilon t_\varepsilon{}^\xi + \omega^{\xi\eta} x_\eta\big) \cdot \partial_\xi \det e^{-1}. \quad (42)$$

Before presenting the field strength tensor, we introduce the commutation relation between the translational derivatives:

$$[d_\alpha, d_\beta] = H_{\alpha\beta}{}^\gamma d_\gamma, \quad (43)$$

where $H_{\alpha\beta}{}^\gamma$ is expressed in terms of $e_\alpha{}^\gamma$ as follows:

$$H_{\alpha\beta}{}^\gamma = e^{-1\gamma}{}_\varepsilon \big(e_\alpha{}^\xi \cdot \partial_\xi e_\beta{}^\varepsilon - e_\beta{}^\xi \cdot \partial_\xi e_\alpha{}^\varepsilon\big). \quad (44)$$

Here, $e^{-1\gamma}{}_\varepsilon$ is the inverse matrix of $e_\alpha{}^\gamma$, i.e. $e_\alpha{}^\varepsilon \cdot e^{-1\gamma}{}_\varepsilon = \delta_\alpha{}^\gamma$. In order to obtain the field strength operator, we calculate the commutator of the gauge covariant derivatives:

$$\begin{aligned} S_{\alpha\beta} &= [\tilde\nabla_\alpha, \tilde\nabla_\beta] = [d_\alpha + B_\alpha + C_\alpha, d_\beta + B_\beta + C_\beta] \\ &= H_{\alpha\beta}{}^\gamma d_\gamma - \big(B_{\alpha\beta}{}^\gamma - B_{\beta\alpha}{}^\gamma\big) d_\gamma \\ &\quad + d_\alpha B_\beta - d_\beta B_\alpha + d_\alpha C_\beta - d_\beta C_\alpha + [B_\alpha, B_\beta] \\ &\quad + [C_\alpha, B_\beta] + [C_\alpha, C_\beta]. \end{aligned} \quad (45)$$

The expressions for the commutators entering in (45) are

$$\begin{aligned} &[B_\alpha, B_\beta] = -\frac{i}{4} B_{\alpha\varepsilon}{}^\gamma B_\beta{}^{\delta\epsilon} \Sigma_{\gamma\delta} + \frac{i}{4} B_\alpha{}^{\delta\varepsilon} B_{\beta\varepsilon}{}^\gamma \Sigma_{\gamma\delta}, \\ &[C_\alpha, B_\beta] = i\lambda B_\alpha{}^\delta B_{\beta\delta}{}^\gamma \Sigma_{\gamma 5}, \\ &[B_\alpha, C_\beta] = -i\lambda B_{\alpha\varepsilon}{}^\gamma B_\beta{}^\varepsilon \Sigma_{\gamma 5}, \\ &[C_\alpha, C_\beta] = 2i\lambda^2 B_\alpha{}^\gamma B_\beta{}^\delta \Sigma_{\gamma\delta}. \end{aligned} \quad (46)$$

Now, defining the tensor coefficients of d_γ,

$$T_{\alpha\beta}{}^\gamma = B_{\alpha\beta}{}^\gamma - B_{\beta\alpha}{}^\gamma - H_{\alpha\beta}{}^\gamma, \quad (47)$$

we can rewrite $S_{\alpha\beta}$ as

$$S_{\alpha\beta} = -T_{\alpha\beta}{}^\gamma \tilde\nabla_\gamma + \frac{i}{4} \tilde R^{\gamma\delta}{}_{\alpha\beta} \Sigma_{\gamma\delta} + \frac{i}{4} \tilde R^{\gamma 5}{}_{\alpha\beta} \Sigma_{\gamma 5}, \quad (48)$$

where $\tilde R^{\gamma\delta}{}_{\alpha\beta}$ and $\tilde R^{\gamma 5}{}_{\alpha\beta}$ have the form

$$\begin{aligned} \tilde R^{\gamma\delta}{}_{\alpha\beta} &= d_\alpha B_\beta{}^{\gamma\delta} - d_\beta B_\alpha{}^{\gamma\delta} + B_\alpha{}^{\delta\varepsilon} B_{\beta\varepsilon}{}^\gamma \\ &\quad - B_\beta{}^{\delta\varepsilon} B_{\alpha\varepsilon}{}^\gamma - H_{\alpha\beta}{}^\varepsilon B_\varepsilon{}^{\gamma\delta} \\ &\quad + 4\lambda^2 \big(B_\alpha{}^\gamma B_\beta{}^\delta - B_\alpha{}^\delta B_\beta{}^\gamma\big) \end{aligned} \quad (49)$$

and

$$\begin{aligned} \tilde R^{\gamma 5}{}_{\alpha\beta} &= -4\lambda \big(d_\alpha B_\beta{}^\gamma - d_\beta B_\alpha{}^\gamma\big) \\ &\quad - 4\lambda \big(B_{\alpha\varepsilon}{}^\gamma B_\beta{}^\varepsilon - B_{\beta\delta}{}^\gamma B_\alpha{}^\delta - H_{\alpha\beta}{}^\varepsilon B_\varepsilon{}^\gamma\big) \\ &= -4\lambda \big(\tilde\nabla_\alpha B_\beta{}^\gamma - \tilde\nabla_\beta B_\alpha{}^\gamma\big) + 4\lambda H_{\alpha\beta}{}^\varepsilon B_\varepsilon{}^\gamma, \end{aligned} \quad (50)$$

respectively. Next, we determine the local infinitesimal transformations of $H_{\alpha\beta}{}^\gamma$, $T_{\alpha\beta}{}^\gamma$ and $\tilde R^{\gamma\delta}{}_{\alpha\beta}$:

$$\begin{aligned} \delta H_{\alpha\beta}{}^\gamma &= -\big(\varepsilon^\rho + \lambda^2 \varepsilon^\mu t_\mu{}^\rho + \omega^{\rho\eta} x_\eta\big) \cdot \partial_\rho H_{\alpha\beta}{}^\gamma \\ &\quad + \omega_\alpha{}^\rho H_{\rho\beta}{}^\gamma + \omega_\beta{}^\rho H_{\alpha\rho}{}^\gamma + \omega^\gamma{}_\xi H_{\alpha\beta}{}^\xi \\ &\quad + e_\alpha{}^\xi \cdot \partial_\xi \omega_\beta{}^\gamma - e_\beta{}^\xi \cdot \partial_\xi \omega_\alpha{}^\gamma, \end{aligned} \quad (51)$$

$$\begin{aligned} \delta T_{\alpha\beta}{}^\gamma &= -\big(\varepsilon^\xi + \lambda^2 \varepsilon^\varepsilon t_\varepsilon{}^\xi + \omega^{\xi\eta} x_\eta\big) \cdot \partial_\xi T_{\alpha\beta}{}^\gamma \\ &\quad + \omega_\alpha{}^\xi T_{\xi\beta}{}^\gamma + \omega_\beta{}^\xi T_{\alpha\xi}{}^\gamma + \omega^\gamma{}_\xi T_{\alpha\beta}{}^\xi \end{aligned} \quad (52)$$

and

$$\begin{aligned} \delta \tilde R^{\gamma\delta}{}_{\alpha\beta} &= -\big(\varepsilon^\xi + \lambda^2 \varepsilon^\varepsilon t_\varepsilon{}^\xi + \omega^{\xi\eta} x_\eta\big) \cdot \partial_\xi \tilde R^{\gamma\delta}{}_{\alpha\beta} \\ &\quad + \omega_\alpha{}^\xi \tilde R^{\gamma\delta}{}_{\xi\beta} + \omega^\xi{}_\beta \tilde R^{\gamma\delta}{}_{\alpha\xi} \\ &\quad + \omega^\gamma{}_\xi \tilde R^{\xi\delta}{}_{\alpha\beta} + \omega^\delta{}_\xi \tilde R^{\gamma\xi}{}_{\alpha\beta}. \end{aligned} \quad (53)$$

In all these calculations we have worked on Minkowski space-time, $(\mathcal{R}^4, \eta)$. If we consider an indefinite metric tensor

$$g^{\mu\nu} = e_\alpha{}^\mu e^{\alpha\nu} \quad (54)$$

and the corresponding Riemannian manifold $(\mathcal{R}^4, g)$ endowed with an arbitrary base $\hat e_\alpha$, then we can define the commutation coefficients $c_{\alpha\beta}{}^\gamma$ (structure functions) [8]

$$[\hat e_\alpha, \hat e_\beta] = c_{\alpha\beta}{}^\gamma \hat e_\gamma. \quad (55)$$

In our case $\hat e_\alpha = d_\alpha$ and $c_{\alpha\beta}{}^\gamma = H_{\alpha\beta}{}^\gamma$. The connection coefficients can then be identified with the gauge fields as $\Gamma^\gamma{}_\alpha{}^\delta = -B_\alpha{}^{\gamma\delta}$. In our model, $T_\alpha{}^{\gamma\delta}$ and $\tilde R^{\gamma\delta}{}_{\alpha\beta}$ are the torsion and curvature tensors, respectively, and

$$R^{\gamma 5}{}_{\alpha\beta} \equiv 4\lambda T_{\alpha\beta}{}^\gamma. \quad (56)$$

6 De Sitter gauge invariant matter actions. Scalar, spinor and vector fields

As we mentioned in Sect. 3, the relation (26) is not yet sufficient for the original action to be locally de Sitter gauge invariant. We have to complete the Lagrangian density with another term in order to obtain a pure divergence. Using the transformation law for $\det e^{-1}$, we consider the combination

$$\mathcal{L}_\mathrm{M}(\varphi_j, \tilde\nabla_\alpha \varphi_j) \to \det e^{-1} \cdot \mathcal{L}_\mathrm{M}(\varphi_j, \tilde\nabla_\alpha \varphi_j). \quad (57)$$

Under local de Sitter gauge transformations it becomes

$$\begin{aligned} &\det e'^{-1} \mathcal{L}_\mathrm{M}(\varphi'_j, \tilde\nabla'_\alpha \varphi'_j) \\ &\quad = \det e^{-1} \mathcal{L}_\mathrm{M}(\varphi_j, \tilde\nabla_\alpha \varphi_j) \\ &\qquad - \partial_\gamma \big(\big(\varepsilon^\gamma + \lambda^2 \varepsilon^\xi t_\xi{}^\gamma + \omega^{\gamma\delta} x_\delta\big) \det e^{-1} \mathcal{L}_\mathrm{M}(\varphi_j, \tilde\nabla_\alpha \varphi_j)\big). \end{aligned} \quad (58)$$

Therefore, the minimally extended de Sitter gauge invariant matter action can be written as

$$S_{\mathrm{M}} = \int \mathrm{d}^4x \det e^{-1}(x) \cdot \mathcal{L}_{\mathrm{M}}\big(\varphi_j(x), \tilde{\nabla}_\alpha \varphi_j(x)\big). \tag{59}$$

The action for a massive scalar field $\varphi(x)$ can be extended to a locally de Sitter gauge invariant form:

$$S_{\mathrm{M}} = \int \mathrm{d}^4x \det e^{-1} \left\{ \frac{1}{2} d_\alpha \varphi \cdot d^\alpha \varphi - \frac{1}{2} m^2 \varphi^2 \right\}. \tag{60}$$

For Dirac spinor and massive vector fields, the action has the form [3, 7]

$$\begin{aligned} S_{\mathrm{M}} = {} & \int \mathrm{d}^4x \det e^{-1} \left\{ \frac{i}{2} \overline{\psi} \gamma^\alpha (\tilde{\nabla}_\alpha \psi) - \frac{i}{2} \overline{(\tilde{\nabla}_\alpha \psi)} \gamma^\alpha \psi \right\} \\ & - m \int \mathrm{d}^4x \det e^{-1} \overline{\psi} \psi \end{aligned} \tag{61}$$

and

$$S_{\mathrm{M}} = \int \mathrm{d}^4x \det e^{-1} \left\{ -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{2} m^2 A_\alpha A^\alpha \right\}, \tag{62}$$

respectively, where $F_{\alpha\beta} = \tilde{\nabla}_\alpha A_\beta - \tilde{\nabla}_\beta A_\alpha$.

7 Regularization technique with ζ function

In this section we determine the heat kernel coefficients c_1 and c_2 belonging to a general hermitean second order differential operator M defined on the Minkowski space-time $(\mathcal{R}^4, \eta)$. We introduce the operator M

$$M = -D_\alpha D^\alpha + E, \quad D_\alpha = \nabla_\alpha + A_\alpha. \tag{63}$$

The heat kernel $K(is; x, y)$, $s > 0$, belonging to M_x fulfills

$$\left(\frac{\partial}{\partial(is)} + M_x \right) K(is; x, y) = 0, \tag{64}$$

together with the initial condition [7, 12, 13]

$$\lim_{s \to 0} K(is; x, y) = \frac{1}{\det e^{-1}} \delta(x - y). \tag{65}$$

In (64) the differential operator M acts on the heat kernel K and the index x denotes the derivative of the heat kernel with respect to x. For $y \to x$ and $s \to 0$, we consider the small s expansion of $K(is; x, y)$:

$$K(is; x, y) \sim \frac{i}{(4\pi i s)^{\frac{d}{2}}} \mathrm{e}^{-\frac{r^2(x,y)}{4is}} \sum_{k=0}^{\infty} (is)^k c_k(x, y). \tag{66}$$

Our main task is to evaluate the derivatives of different orders for the function $r^2(x)$ and the coefficient functions $c_k(x, y)$. By a direct but tedious calculation, we get

$$\begin{aligned} & \nabla_\alpha r^2(x) = 0, \\ & \nabla_{\beta\alpha} r^2(x) = 2\eta_{\beta\alpha}, \\ & \nabla_{\gamma\beta\alpha} r^2(x) = 0, \\ & \nabla_{\delta\gamma\beta\alpha} r^2 = \frac{2}{3}(R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\beta\delta}), \end{aligned} \tag{67}$$

and the coefficients c_1 and c_2:

$$c_1(x) = -\frac{1}{6} R^{\alpha\beta}{}_{\alpha\beta} - E, \tag{68}$$

$$\begin{aligned} c_2(x) = {} & -\frac{1}{30} \nabla_\gamma{}^\gamma R^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{72} R_{\alpha\beta}{}^{\alpha\beta} \cdot R_{\gamma\delta}{}^{\gamma\delta} \\ & + \frac{1}{180} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - \frac{1}{180} R_{\alpha\gamma}{}^\alpha{}_\delta \cdot R_\beta{}^{\gamma\beta\delta} \\ & + \frac{1}{12} F_{\alpha\beta} \cdot F^{\alpha\beta} + \frac{1}{6} R^{\alpha\beta}{}_{\alpha\beta} \cdot E \\ & - \frac{1}{6} \big[D_\alpha, [D^\alpha, E]\big] + \frac{1}{2} E^2, \end{aligned} \tag{69}$$

where in (69) we have $F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha + [A_\alpha, A_\beta]$. We remark that the tensor $R^{\gamma\delta}_{\alpha\beta}$ now contains the parameter λ in each of its components, i.e. the cosmological constant will automatically be included in the expression of the action.

Coming back to the ζ function, we recall the well known property

$$\ln \det M \equiv -\lim_{u \to 0} \frac{\mathrm{d}}{\mathrm{d}u} \zeta(u; \mu; M). \tag{70}$$

If we consider the behavior of the functional determinant under a change of scale, $\tilde{\mu} = \lambda\mu$, then we have

$$\zeta'(0; \tilde{\mu}; M) = \zeta'(0; \mu; M) + 2 \ln \lambda \cdot \zeta(0; \mu; M). \tag{71}$$

This result shows that the change of the functional determinant under rescaling is fully determined by $\zeta(0; \mu; M)$. For a d-dimensional Minkowski space-time, the function $\zeta(0; \mu; M)$ has the form

$$\zeta(0; \mu; M) = \frac{i}{(4\pi)^{d/2}} \int \mathrm{d}^d x \det e^{-1} \operatorname{tr} c_{d/2}(x). \tag{72}$$

If we turn back to the action written in the previous section, then we can construct the functional integral [11, 16, 17] for different types of fields. The simplest case is the scalar field, for which the functional integral is

$$Z_\varphi[e] = \int \mathcal{D}\varphi \mathrm{e}^{i S_{\mathrm{M}}(\varphi; e)}. \tag{73}$$

After integration by parts we can rewrite the action as a scalar product:

$$S_{\mathrm{M}}(\varphi; e) = \frac{1}{2}\big(\varphi, M_\varphi(e)\varphi\big)_e. \tag{74}$$

Introducing the second order operator M_φ

$$M_\varphi(e) = -\nabla_\alpha \nabla^\alpha - m^2, \tag{75}$$

and performing a Gaussian integration, we can write

$$Z_\varphi[e] = \mathrm{e}^{-\frac{1}{2}\ln\det M_\varphi(e)}. \tag{76}$$

We are interested in the behavior of $Z_\varphi[e]$ under rescaling using the ζ function. At the scale μ, we have

$$Z_\varphi[\mu; e] = \mathrm{e}^{\frac{1}{2}\zeta'(0;\mu;M_\varphi(e))}. \tag{77}$$

If we consider a new scale $\tilde{\mu} = \lambda\mu$, we get

$$Z_\varphi[\tilde{\mu}; e] = Z_\varphi[\mu; e]\mathrm{e}^{\ln\lambda\cdot\zeta(0;\mu;M_\varphi(e))}. \tag{78}$$

In (73)–(78) the index φ denotes the reference to a scalar field $\varphi(x)$. In the same way, we can construct the second order operator M and the functional integral for the spinor and vector fields.

8 Concluding remarks

Based on the conception of de Sitter symmetry as a pure inner symmetry, we developed a gauge theory of gravitation. The gravitational interaction is mediated by gauge fields defined on a fixed Minkowski space-time. Because the conserved current J^γ has the same value for the global de Sitter symmetry as well as for the pure inner symmetry, the two complementary conceptions are equivalent, and they lead to similar physical consequences. Replacing the usual partial derivative ∂_α with the covariant derivative $\tilde{\nabla}_\alpha$, we introduced the gauge fields and studied in detail their transformation laws. Various quantities, such as the curvature and torsion tensors, depend on the deformation parameter λ. In the limit $\lambda \to 0$, a process called contraction, all results obtained for the de Sitter group pass into those obtained for the Poincaré group. The field dynamics has been determined by imposing consistency requirements in accord to the renormalization properties of matter fields on gravitational backgrounds. We have also presented a renormalization technique using the ζ function. To use this technique we defined first the second order differential operator M, and then we calculated the coefficients c_1 and c_2 belonging to this operator. Using these coefficients we determined the ζ function, $\zeta(0; \mu; M)$. Its insertion into the functional integral gives anomalous terms in scalar, spinor or vector fields. At this point we remark once more that our investigations have been made on a space with null torsion. Finally, using the obtained results, we can construct a minimal action for the gauge fields defined on a fixed Minkowski space-time $(\mathcal{R}^4, \eta)$. This action is invariant on the one hand under local de Sitter gauge transformations, and on the other hand under global de Sitter transformations.

Acknowledgement The authors acknowledge the support by the CNCSIS-UEFISCSU grant 620 of the Minister of Education, Research and Youth of Romania.

References

1. P.A.M. Dirac, Ann. Math. **36**, 657 (1935)
2. S.L. Adler, Phys. Rev. D **6**, 3445 (1972)
3. R. Aldrovandi, J.G. Pereira, arXiv:gr-qc/9610068 (1996)
4. R. Aldrovandi, J.G. Pereira, arXiv:gr-qc/9809061 (1998)
5. R. Banerjee, Ann. Phys. **322**, 2129 (2007), arXiv:hep-th/0608045
6. R. Banerjee, B.R. Majhi, Ann. Phys. **323**, 705 (2008), arXiv:hep-th/0703207
7. C. Wiesendanger, Class. Quant. Grav. **13**, 681 (1996), arXiv:gr-qc/9505049
8. M. Blagojević, *Gravitation and Gauge Symmetries* (Institute of Physics Publishing, Bristol, 2002)
9. R. Banerjee, S. Deguchi, Phys. Lett. B **632**, 579 (2006), arXiv:hep-th/0509161
10. M. Chaichian, N.F. Nelipa, *Introduction to Gauge Field Theories* (Springer, Berlin, 1984)
11. J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 4th edn. (Oxford University Press, London, 2002)
12. D. Bailin, A. Love, *Introduction to Gauge Field Theory* (Institute of Physics Publishing, Bristol, 1993)
13. E. Fradkin, *General Field Theory*. Unpublished
14. G. Zet, V. Manta, S. Babeti, Int. J. Mod. Phys. C **14**, 41 (2003)
15. G. Zet, V. Manta, S. Oancea, I. Radinschi, B. Ciobanu, Math. Comput. Mod. **43**, 458 (2006)
16. M. Chaichian, A. Demichev, *Path Integrals in Physics*, vol. I (Institute of Physics Publishing, Bristol, 2001)
17. M. Chaichian, A. Demichev, *Path Integrals in Physics*, vol. II (Institute of Physics Publishing, Bristol, 2001)