Using the same conditions (15), removing the mixed derivative, from Maxwell equations (12 – 13) can be read the system of equation

$$\frac{\partial^2 A_1}{\partial r^2}+\frac{2}{r}\frac{\partial A_1}{\partial r}-\frac{2}{r^2}A_1-\frac{\partial^2 A_1}{\partial t^2}=-2ek\frac{|N|^2}{r^2}, \tag{17}$$

$$\frac{\partial^2 A_4}{\partial r^2}+\frac{2}{r}\frac{\partial A_4}{\partial r}-\frac{\partial^2 A_4}{\partial t^2}=2e\omega\frac{|N|^2}{r^2}, \tag{18}$$

The equation (18) admits a particular oscillating non-trivial solution of the form (Fig.1):

$$A_1(r,t)=ln(r)\left[2ek\frac{|N|^2}{ln\left(\frac{r^2}{r_0}\right)}+a\,sin\left(\frac{1}{r}\sqrt{\frac{ln\left(\frac{r^2}{r_0}\right)}{ln(r)}}t+\varphi\right)\right] \tag{19}$$

The equation (19) admits in the same manner, a particular non-trivial solution of the form:

$$A_4(r,t)=ln(r)\left[2e\omega|N|^2+b\,exp\left(-\frac{1}{r}\sqrt{\frac{t^2}{ln(r)}}\right)\right] \tag{20}$$

Considering these first order solutions, could be evaluated the electric charge density, which has the expression

$$\rho=e^2N^4\omega\left[2e\omega ln(r)eN^2-eb\,exp\left(-\frac{1}{r}\sqrt{\frac{t^2}{ln(r)}}\right)ln(r)+\omega\right] \tag{21}$$

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Renormalization In Quantum Gauge Theory Using Zeta-Function Method

Viorel Chiritoiu[a] and Gheorghe Zet[b]

[a] *"Politehnica" University Timisoara, Technical Physics Department, Timisoara, Romania*
[b] *"Gh. Asachi" Technical University, Department of Physics, Iasi, Romania*

Abstract. It is possible to consider space-time symmetries (for example Poincaré or de Sitter) as purely inner symmetries. A formulation of the de Sitter symmetry as purely inner symmetry defined on a fixed Minkowski space-time is presented. We define the generators of the de Sitter group and write the equations of structure using a constant deformation parameter λ. Local gauge transformations and corresponding covariant derivative depending on gauge fields are obtained. The method of generalized zeta-function is used to realize the renormalization. An effective integral of action is obtained and a comparison with other results is given.

Keywords: De Sitter pure inner symmetry, hyperbolic fluctuation operator, zeta-function
PACS: 11.10.Gh; 11.10.Ef; 11.15.-q

INTRODUCTION

Most of the existing gauge theories of gravitation adopt a geometrical description of gravity. Namely, the Poincaré group is considered partly as a space-time partly as an internal symmetry group. The local extension of its space-time part becomes then the diffeomorphism group and the gauge theory is invariant under general coordinate transformations and local Lorentz frame rotations. Therefore, this local symmetry group is connected with the geometry of the space-time.

It is possible also to consider space-time symmetries (for example Poincaré or de Sitter in this paper) as purely inner symmetries [1, 2, 3]. This leads to a description of the gauge theory of gravitation which is in a complete analogy with the description of inner symmetries as groups of generalized "rotations" in field space.

In this paper we consider the group de Sitter (DS) as purely inner symmetry and develop a gauge theory of gravitation. We obtain an effective integral of action which automatically includes the cosmological constant. The method of generalized zeta-function is used to study the renormalization of the theory.

In Section 2 we introduce the DS gauge group and give in an explicitly form its equation of structures. The gauge covariant derivative is introduced as usually, considering the DS group as an internal symmetry and introducing the corresponding gauge fields. The strength field is defined as the commutator of two gauge covariant derivatives.

The renormalizability of the theory is studied in Section 3, using the method of generalized zeta function. The change of the partition function with respect to scale

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transform is calculated for the case of a spinor Dirac field interacting with the gravitational field described by the gauge potentials. Then, a minimal field gauge action, compatible with renormalizability requirements and including the cosmological constant, is determined.

Finally, some concluding remarks are given in the Section 4. It is emphasized that in our model there is no any direct interrelation between gravity and the structure of space-time. At quantum level it may conceptually be easier to deal with a field theoretical description of gravitation free of any geometrical aspects.

DE SITTER GAUGE THEORY

We consider a gauge theory of gravitation having de Sitter (DS) group as local symmetry. Let X_A, $A=1,2,...,10$ be a basis of DS Lie algebra with the corresponding equations of structure given by [2]

$$[X_A, X_B] = i\, f_{AB}^{\ \ C} X_C, \tag{1}$$

where $f_{AB}^{\ \ C}$ are the constants of structure whose form are given bellow in Eq. (3).

In order to write the constant of structures $f_{AB}^{\ \ C}$ in a compact form, we use the following notations for the index A:

$$A = \begin{cases} a & = 0,1,2,3 \\ [\mathrm{ab}] & = [01],[02],[03],[12],[13],[23] \end{cases}. \tag{2}$$

This means that A can stand for a single index like 2 as well as for a pair of indices like [01], [12], etc. The infinitesimal generators X_A are interpreted as: $X_A = P_a$ (energy-momentum operators) and $X_{[ab]} = M_{ab}$ (angular momentum operators) with the property $M_{ab} = -M_{ba}$. The constants of structure $f_{AB}^{\ \ C}$ in (1) have the expressions:

$$\begin{cases} f_{bc}^{\ \ a} & = f_{c[de]}^{\ \ \ ab} = f_{[bc][de]}^{\ \ \ \ a} = 0 \\ f_{cd}^{\ \ [ab]} & = 4\lambda^2\left(\delta_c^{\ b}\delta_d^{\ a} - \delta_c^{\ a}\delta_d^{\ b}\right) \\ f_{b[cd]}^{\ \ \ a} & = -f_{[cd]b}^{\ \ \ a} = \frac{1}{2}\left(\eta_{bc}\delta_d^{\ a} - \eta_{bd}\delta_c^{\ a}\right) \\ f_{[ab][cd]}^{\ \ \ \ [ef]} & = \frac{1}{4}\left(\eta_{bc}\delta_a^{\ e}\delta_d^{\ f} - \eta_{ac}\delta_b^{\ e}\delta_d^{\ f} + \eta_{ad}\delta_b^{\ e}\delta_c^{\ f} - \eta_{bd}\delta_a^{\ e}\delta_c^{\ f}\right) - e \leftrightarrow f \end{cases}, \tag{3}$$

where λ is a real parameter, and $\eta_{ab} = diag(1,-1,-1,-1)$ is the Minkowski metric of the space-time. In fact, here we have a deformation of the de Sitter Lie algebra having λ as parameter. Considering the contraction $\lambda \to 0$ we obtain the Poincaré Lie algebra, i.e., the group DS contracts to the Poincaré group.

Now we introduce the local DS gauge transformation and the corresponding gauge covariant derivative ∇_a, considering DS as an internal group of symmetry. As usually in any gauge theory, we have

$$\nabla_a = \partial_a + B_a, \tag{4}$$

together with the following decomposition of B_a with respect to the infinitesimal generators P_a and M_{ab}

$$B_a = -iB_a^{\ b}\cdot P_b + \frac{1}{2}B_a^{\ bc}\cdot M_{bc}. \tag{5}$$

The corresponding generators of the DS group in the field space have the form:

$$P_a = i\partial_a + \lambda K_a, \qquad M_{ab} = i\left(x_a\partial_b - x_b\partial_a\right) + \frac{1}{2}\Sigma_{ab}, \tag{6}$$

where K_a are the "translation" de Sitter generators and Σ_{ab} the spin angular momentum operators. The last one (Σ_{ab}) satisfy commutation relations of the same form as M_{ab} and K_a have the expression [3]:

$$K_a = i\left(2\eta_{ab}x^b x^c - \sigma^2\delta_a^{\ c}\right)\partial_c, \qquad \sigma^2 = \eta_{ab}x^a x^b. \tag{7}$$

We also can decompose B_a with respect to ∂_a and Σ_{ab} as follows:

$$B_a = \left[B_a^{\ b} + \lambda B_a^{\ d}\left(2\eta_{dc}x^c x^b - \sigma^2\delta_d^{\ b}\right)\right]\partial_b + \frac{i}{4}B_a^{\ bc}\Sigma_{bc}. \tag{8}$$

Introducing (8) into Eq. (4) and denoting

$$e_a^{\ b} = \delta_a^{\ b} + \lambda B_a^{\ d}\left(2\eta_{dc}x^c x^b - \sigma^2\delta_d^{\ b}\right) + B_a^{\ bc}x_c, \tag{9}$$

we obtain

$$\nabla_a = e_a^{\ b}\partial_b + \frac{i}{4}B_a^{\ bc}\Sigma_{bc}. \tag{10}$$

Because in our model the coordinate and DS gauge transformations are strictly separated, we emphasize that the introduction of $B_a^{\ b}, B_a^{\ bc}$ and $e_a^{\ b}$ gauge fields has no implications on the structure of the underlying space-time, which is assumed to be (M_4, η) endowed with the Minkowski metric η.

Abbreviating

$$d_a = e_a^{\ b}\partial_b, \qquad B_a = \frac{i}{4}B_a^{\ bc}\Sigma_{bc}, \tag{11}$$

where Σ_{ab} must be considered into the Lorentz group representation it acts on, we can write the gauge covariant derivative (10) under the simple form:

$$\nabla_a = d_a + B_a. \tag{12}$$

The derivative d_a can be just considered as a translation gauge covariant derivative [4]. In order to obtain the tensor (field strength operator) F_{ab} of the gauge fields, we introduce the non-covariant decomposition

$$[d_a, d_b] = H_{ab}^{\ \ c} d_c. \tag{13}$$

The quantity $H_{ab}^{\ \ c}$ is expressed in terms of $e_a^{\ b}$ as:

$$H_{ab}^{\ \ c} = \bar{e}_m^{\ c}\left(e_a^{\ d}\partial_d e_b^{\ m} - e_b^{\ d}\partial_d e_a^{\ m}\right), \tag{14}$$

where $\bar{e}_m^{\ c}$ is the matrix inverse of $e_c^{\ n}$, i.e. $\bar{e}_m^{\ c} e_c^{\ n} = \delta_m^{\ n}$. Using the definition of the field strength operator in a gauge theory, we have:

$$F_{ab} = [\nabla_a, \nabla_b] = H_{ab}^{\ \ c}d_c - \left(B_{ab}^{\ \ c} - B_{ba}^{\ \ c}\right)d_c + d_a B_b - d_b B_a + [B_a, B_b]. \tag{15}$$

If we introduce the tensor

$$T_{ab}{}^{c} = B_{ab}{}^{c} - B_{ba}{}^{c} - H_{ab}{}^{c}, \tag{16}$$

then we can rewrite F_{ab} as

$$F_{ab} = -T_{ab}{}^{c}\nabla_c + \frac{i}{4}R^{cd}{}_{ab}\Sigma_{cd}, \tag{17}$$

where $R^{cd}{}_{ab}$ has the expression

$$R^{cd}{}_{ab} = d_a B_b{}^{cd} - d_b B_a{}^{cd} + B_a{}^{de} B_{be}{}^{c} - B_b{}^{de} B_{ae}{}^{c} - H_{ab}{}^{e} B_e{}^{cd}. \tag{18}$$

In what follows we will use the shorthand notation

$$R_{ab} = \frac{i}{4}R^{cd}{}_{ab}\Sigma_{cd}. \tag{19}$$

As F_{ab} in (17) has decomposition with respect to ∇_a and Σ_{cd} it acts in general not only as a matrix but also as a first order differential operator in field space. But, if we suppose that

$$H_{ab}{}^{c} = B_{ab}{}^{c} - B_{ba}{}^{c}, \tag{20}$$

that is we take $T_{ab}{}^{c} = 0$, then we can write Eq. (15) under the form:

$$F_{ab} = \frac{i}{4}R^{cd}{}_{ab}\Sigma_{cd} = R_{ab}. \tag{21}$$

We can verify that $T_{ab}{}^{c}$ and $R^{cd}{}_{ab}$ transform homogeneous under infinitesimal local DS gauge transformations. Then, as a consequence, the choice $T_{ab}{}^{c} = 0$ is indeed a gauge covariant statement as an implicitly assumed above.

RENORMALIZATION

In order to analyze the renormalization of our DS gauge theory, we will consider first the globally DS invariant action for a Dirac spinor field (matter field):

$$S_D = \int d^4x \left[\frac{i}{2}\overline{\psi}\gamma^a(\partial_a\psi) - \frac{i}{2}(\partial_a\overline{\psi})\gamma^a\psi - m\overline{\psi}\psi\right]. \tag{22}$$

Then, if we want to obtain a gauge (local) invariant action, we have to change the usual derivative ∂_a in (22) by the gauge covariant derivative defined in Eq. (12):

$$S_D = \int d^4x\, e^{-1}\left[\frac{i}{2}\overline{\psi}\gamma^a(\nabla_a\psi) - \frac{i}{2}(\nabla_a\overline{\psi})\gamma^a\psi - m\overline{\psi}\psi\right]. \tag{23}$$

and to use the new volume element $d^4x\, e^{-1}$, where $e^{-1} = \det\left(\overline{e}_a{}^{b}\right)$. Then, partially integrating ∇_a in the second term of (23), we obtain the form of the Dirac action:

$$S_D = \int d^4x\, e^{-1}\overline{\psi}\left\{i\gamma^a\left(\nabla_a - \frac{1}{2}T_{ba}{}^{b}\right) - m\right\}\psi, \tag{24}$$

The assumption that the interaction of the DS gauge fields with the matter fields (in our case with the Dirac field) is renormalizable, imposes strong conditions on the classical gauge field dynamics. Namely, we know that the change of the partition function of the whole system under rescaling can be absorbed in its classical action yielding at most a nontrivial scale dependence of the different couplings, masses and wave function normalizations. As a consequence, the change of one-loop matter partition under rescaling will allow us to constrain the classical gauge field dynamics. The contribution of the Dirac field to the partition function is given by the following functional integral [5]:

$$Z_\psi(e,B) = \int D\overline{\psi}\, D\psi\, e^{iS_D(\overline{\psi},\psi;e,B)}. \tag{25}$$

Then, we may perform a formal Grassmann integral in (25) and obtain:

$$Z_\psi(e,B) = e^{\frac{1}{2}\ln\det M_\psi(e,B)}, \tag{26}$$

where

$$M_\psi(e,B) = -D_a D^a + \frac{i}{2}R_{ab}\Sigma^{ab} - m^2. \tag{27}$$

Here, $M_\psi(e,B)$ is named hyperbolic fluctuation operator and its expression in (27) is obtain as usually [5] by squaring the Dirac operator introduced in Eq. (24). For the case $T_{ab}{}^{c} = 0$ we consider here, the operator D_a in Eq. (27) is given by the formula:

$$D_a = \nabla_a + B_a. \tag{28}$$

The gauge field (Lie algebra valued) shall only act on the spinor indices and the covariant derivative ∇_a only on vector indices.

The contribution to the partition function normalized at scale μ is given by [6]:

$$Z_\psi(\mu;e,B) = e^{-\frac{1}{2}\zeta'(0;\mu;M_\psi(e,B))}, \tag{29}$$

where $\zeta(s;\mu;M_\psi(e,B))$ is the generalized zeta function of parameter s associated to the hyperbolic fluctuation operator $M_\psi(e,B)$ and $\zeta'(0;\mu;M_\psi(e,B))$ is the derivative of the generalized zeta function with respect to s taken for $s = 0$.

We just remember the fact that zeta function is given by

$$\zeta(0;\mu;M_\psi(e,B)) = \frac{i}{(4\pi)^2}\int d^4x \det e^{-1}\, \mathrm{tr}\, c_2(x) \tag{30}$$

and the coefficient function $c_2(x)$ for the Dirac field in the case T = 0 has the form

$$\mathrm{tr}\, c_2(x) = \frac{1}{30}\nabla_c{}^{c}R^{ab}{}_{ab} + \frac{1}{72}R_{ab}{}^{ab}\cdot R_{cd}{}^{cd} - \frac{7}{360}R_{abcd}\cdot R^{abcd} - \frac{1}{45}R_{ac}{}^{a}{}_{d}\cdot R_b{}^{cbd} + \frac{1}{3}m^2\cdot R^{ab}{}_{ab} + 2m^4. \tag{31}$$

We consider now a new scale $\tilde{\mu} = \lambda\mu$ and determine the corresponding change of $Z_\psi(\mu;e,B)$. To end this, we use the very well known property [6]

$$\zeta'(0;\tilde{\mu};M_\psi(e,B)) = \zeta'(0;\mu;M_\psi(e,B)) + 2\ln\lambda\cdot\zeta(0;\mu;M_\psi(e,B)). \tag{32}$$

Then, we obtain:

$$Z_\psi(\tilde{\mu};e,B) = Z_\psi(\mu;e,B)\cdot e^{-\ln\lambda\cdot\zeta(0;\mu;M_\psi(e,B))}. \tag{33}$$

Finally, we have to evaluate the zeta function yielding the rescaling change in terms of the DS gauge fields and then to determine a minimal field gauge action compatible with renormalizability requirements. Renormalizability of any theory, including dynamical gauge fields, requires that these contributions to the partition function like (33) be expressed as local DS gauge invariant polynomials in the fields $e_a^{\ b}$ and $B_a^{\ bc}$. In our case, under the constraint $T_{ab}^{\ c} = 0$, we obtain as minimal classical action for the gauge fields [7]:

$$S_g(e,B) = \int d^4x\, e^{-1}\left(-\frac{1}{16\pi G}(R - 2\Lambda) + \alpha R^2 + \beta R_{ac}^{\ \ a}{}_d R_b^{\ cbd} + \gamma R_{abcd}R^{abcd}\right) \quad (34)$$

Here, G is the gravitational constant and α, β, γ are the coupling constants. We can see that the DS gauge group automatically enforces a cosmological constant which in our model is equal to $\Lambda = -12\lambda^2$, where λ is the deformation parameter of the de Sitter Lie algebra. We emphasize that S_g in (34) is an action for gauge fields defined on the Minkowski space-time (M_4, η) and is invariant on one hand under local DS gauge transformations, on the other hand under global Poincaré symmetry reflecting the symmetry of the underlying space-time.

CONCLUSIONS

Based on the hypothesis that DS is a purely inner symmetry we have developed a gauge theory of gravitation with the constant cosmological automatically included. When the deformation parameter $\lambda \to 0$, we obtain the Poincaré gauge theory on the Minkowski space-time which do not include the cosmological constant. The gravitational interaction is mediated by gauge fields defined on a fixed Minkowski space-time. Their dynamics has been determined imposing consistency requirements with renormalization properties of matter fields in the gravitational backgrounds. In our model there is no any direct interrelation between gravity and the structure of space-time. At quantum level it may conceptually be easier to deal with a field theoretical description of gravitation free of any geometrical aspects.

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Excitation on the Coherent States of Pseudoharmonic Oscillator

Dusan Popov[a], Nicolina Pop[a], Vjekoslav Sajfert[b]

[a]*Univ. "Politehnica", Dept. of Phys. Foundations of Eng., Bd. Vasile Parvan No. 2, 300223 Timişoara*
[b]*Univ. of Novi Sad, Technical Faculty "M. Pupin", Djure Djakovica bb, 23000 Zrenjanin, Serbia*

Abstract. In the last decades, much attention has been paid to the excitation on coherent states, especially for coherent states of the harmonic oscillator ([1] and references therein). But an interesting anharmonic oscillator with many potential applications is also the pseudoharmonic oscillator (PHO). So, in the present paper we have defined the excitation on the Klauder-Perelomov coherent states (E-KP-CSs) for the PHO. These states are obtained by repeatedly operating the raising operator K_+ on a usual Klauder-Perelomov coherent state (KP-CS) of the PHO [2]. We have verified that really, the E-KP-CSs fulfill all the properties of the coherent states, as stated by Klauder [3]. We have examined the nonclassical properties of the E-KP-CSs, by using the density matrix formalism and examining the dependence of the Mandel parameter $Q_{z,k;m}(|z|^2)$ on the $|z|^2$ and on the m. It seems that these states can be used in optical communication field and in the physics of quantum information, as signal beams, due to the fact that in these fields the nonclassicality plays an important role.

Keywords: coherent states, pseudoharmonic oscillator, density matrix.
PACS: 03.65 w, 03.65 Db, 34.20 cf.

PSEUDOHARMONIC OSCILLATOR (PHO)

The pseudoharmonic oscillator (PHO) [4, 5] is an anharmonic potential, which, like the harmonic oscillator (HO) potential, also allows an exact mathematical treatment. This potential may be considered in a certain sense as an intermediate oscillator between the HO and more anharmonic oscillators, e. g. Morse oscillator (MO), Pöschl-Teller oscillator (PTO) (which are more realistic).

The effective potential of PHO [2] is:

$$V_J(r) = \frac{m\omega^2}{8} r_J^{\ 2}\left(\frac{r}{r_J} - \frac{r_J}{r}\right)^2 + \frac{m\omega^2}{4}\left(r_J^{\ 2} - r_o^{\ 2}\right), \quad (1)$$

where m is the reduced mass, ω is the angular frequency and r_0 is the equilibrium distance between the diatomic molecule nuclei. The appearing constants are defined as

$$r_J = \left[\frac{2\hbar}{m\omega}\left(\alpha^2 - \frac{1}{4}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}; \qquad \alpha = \left[\left(J + \frac{1}{2}\right)^2 + \left(\frac{m\omega}{2\hbar} r_0^{\ 2}\right)^2\right]^{\frac{1}{2}}$$

where $J = 0,1,2,...$ is the rotational quantum number. When Eq. (1) is compared with the HO potential: $V_{HO} = \frac{1}{2} m\omega^2 (r - r_0)^2 \quad (2)$

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