UNIFIED GAUGE THEORY ON NONCOMMUTATIVE SPACE-TIME

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A gauge theory describing simultaneously different interactions between internal SU(2) gauge and gravitational fields is formulated, choosing the group $SU(2) \times SO(4,1)$ as a local symmetry. Both internal and external (space-time) symmetries are considered. All the fields are described by gauge potentials. A solution of Schwarzschild-Reissner-Nordström-de-Sitter type is obtained first in the commutative space-time. We suppose then that the space-time is noncommutative. The corrections for tetrad fields and metric components are calculated up to the second order in the noncommutativity parameter. The solutions reduce to the deformed Reissner-Nordström or Schwarzschild ones when the cosmological constant and respectively the electric charge of the gravitational source vanish.

1. INTRODUCTION

The gauge theory of gravitation has been considered by many authors in order to describe the gravity in a similar way with other interactions (electromagnetic, weak or strong) [1]. Some authors consider the Poincaré (inhomogeneous Lorentz) group *ISO*(3,1) or de-Sitter *SO*(4,1) group as "active" symmetry groups, *i.e.* acting on the space-time coordinates [2]. Others adopt the "passive" point of view when the space-time coordinates are not affected by group transformations [3, 4]. Only the fields change under the action of the symmetry group.

Although the Poincaré gauge theory leads to a satisfactory classical theory of gravity, the analogy with gauge theories of internal symmetries is not a satisfactory one because of the specific treatment of translations [5]. It is possible, however, to formulate the gauge theory of gravity in a way that treats the whole *ISO*(3,1) in a more unified framework. The approach is based on the *SO*(4,1) group and the Lorentz and translation parts are distinguished through a mechanism of spontaneous symmetry breaking [6]. An immediate consequence of replacing *ISO*(3,1) by the *SO*(4,1) group as the symmetry underlying the Universe is the appearance of a non-vanishing cosmological constant Λ, which is determined by a real parameter λ of deformation. When we consider the limit $\lambda \to 0$, *i.e.* the group contraction process, the de-Sitter group *SO*(4,1) reduces to the Poincaré group *ISO*(3,1), and the corresponding gravitation theory can not describe the cosmological constant [7]. The matter fields are described by an action that is invariant under the global $SO(4,1)$ symmetry and the gravity is introduced as a gauge field in the process of localization of this symmetry.

On the other hands, many recent investigations are oriented towards a formulation of general relativity on noncommutative space-times [18, 19]. In [18], for example, a deformation of Einstein's gravity was studied by gauging the noncommutative $SO(4,1)$ de-Sitter group and using the Seiberg-Witten map [17] with subsequent contraction to the Poincaré group $ISO(3,1)$. In [19, 20] the gravitational gauge potentials for the Schwarzschild and respectively Reissner-Nordström-de-Sitter metrics are calculated.

In this paper, we develop an unified model of the gravitation with other interactions by considering the group $SU(2)\times SO(4,1)$ as gauge symmetry. By contraction to the $ISO(3,1)$ group we can obtain the Poincaré gauge gravity. We obtain first a solution in the commutative case for the gauge potentials and construct a metric of Reissner-Nordström-de-Sitter type. Then, using the Seiberg-Witten map, we calculate the noncommutativity corrections for the gravitational gauge potentials and for the corresponding metric components.

2. THE GAUGE THEORY

The de-Sitter group $SO(4,1)$ has the dimension equal to ten and the $SU(2)$ group is non-abelian, three-dimensional. The infinitesimal generators of the $SO(4,1)$ group are denoted by M_{ab}, a, $b = 0, 1, 2, 3, 4, 5$, and those of $SU(2)$ group by T_α, $\alpha = 1, 2, 3$. The equations of structure have the form [4, 6]:

$$\left[M_{ab}, M_{cd}\right] = \eta_{bc} M_{ad} - \eta_{ac} M_{bd} - \eta_{bd} M_{ac} + \eta_{ad} M_{bc}, \tag{2.1a}$$

$$\left[T_\alpha, T_\beta\right] = \varepsilon_{\alpha\beta\gamma} T_\gamma, \tag{2.1b}$$

$$\left[M_{ab}, T_\alpha\right] = 0, \tag{2.1c}$$

where $\eta_{ab} = (1,-1,-1,-1,-1)$ is the five-dimensional Lorentz metric. A matter field $\phi(x)$ is always referred to a local frame L of the Minkowski space-time. In general, it is a multicomponent object which can be represented as a vector-column. The action of the global de-Sitter group, in the tangent space, transforms an L frame into another L frame and determine an appropriate transformation of the field $\phi(x)$ [4]:

$$\phi'(x') = \left(1 + \frac{1}{2}\lambda^{ab}\Sigma_{ab}\right)\phi(x') \tag{2.2}$$

Here Σ_{ab} are the spin matrices related to the multicomponent structure of $\phi(x)$ and they satisfy the same equations of structure (2.1a) as M_{ab}.

We define now the gauge covariant derivative, associated to the local group of symmetry $SU(2)\times SO(4,1)$:

$$\nabla_\mu \phi(x) = \left(\partial_\mu + \frac{1}{2} A_\mu^{ab} \Sigma_{ab} + A_\mu^\alpha T_\alpha\right)\phi(x), \tag{2.3}$$

where $A_\mu^{ab}(x) = -A_\mu^{ba}(x)$ are the gauge potentials describing the gravitational field and $A_\mu^\alpha(x)$ are the internal gauge potentials associated to the group $SU(2)$. Now, we calculate the commutator $\left[\nabla_\mu, \nabla_\nu\right]$ in order to obtain the expressions of the strength tensors. We have:

$$\begin{aligned}\left[\nabla_\mu, \nabla_\nu\right]\phi(x) = \{&\frac{1}{2}\left[\partial_\mu A_\nu^{ab} - \partial_\nu A_\mu^{ab} + (A_{c\mu}^a A_\nu^{cb} - A_{c\nu}^a A_\mu^{cb})\right]\Sigma_{ab} + \\ &+\left(\partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + \varepsilon^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma\right)T_\alpha\}\phi(x).\end{aligned} \tag{2.4}$$

If we use the general definition

$$\left[\nabla_\mu, \nabla_\nu\right]\phi(x) = \left(\frac{1}{2} F_{\mu\nu}^{ab} \Sigma_{ab} + G_{\mu\nu}^\alpha T_\alpha\right)\phi(x) \tag{2.5}$$

and identify the Eqs. (2.4) and (2.5), we obtain:

$$F_{\mu\nu}^{ab} = \partial_\mu A_\nu^{ab} - \partial_\nu A_\mu^{ab} + \left(A_{c\mu}^a A_\nu^{cb} - A_{c\nu}^a A_\mu^{cb}\right), \tag{2.6}$$

$$G_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + \varepsilon^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma. \tag{2.7}$$

Choosing $a = i,5,\ \ b = j,5,\ \ c = m,5$ with $i, j, m = 0,1,2,3$, and denoting $A_\mu^{i5} = 2\lambda e_\mu^i$, then the Eq. (2.6) becomes:

$$F_{\mu\nu}^{ij} = \partial_\mu A_\nu^{ij} - \partial_\nu A_\mu^{ij} + \left(A_{s\mu}^i A_\nu^{sj} - A_{s\nu}^i A_\mu^{sj}\right) - 4\lambda^2\left(e_\mu^i e_\nu^j - e_\nu^i e_\mu^j\right), \tag{2.8}$$

$$F_{\mu\nu}^i = \partial_\mu e_\nu^i - \partial_\nu e_\mu^i + \left(A_{s\mu}^i e_\nu^s - A_{s\nu}^i e_\mu^s\right). \tag{2.9}$$

In a Riemann-Cartan model the quantities $F_{\mu\nu}^i$ are interpreted as the components of the torsion tensor, and $F_{\mu\nu}^{ij}$ as the components of the curvature tensor associated to the gravitational field whose gauge potentials are $e_\mu^i(x)$ and $A_\mu^{ij}(x)$.

3. MODEL WITH SPHERICAL SYMMETRY

We consider now a particular form of spherically gauge fields of the $SU(2)\times SO(4,1)$ group given by the following ansatz:

$$e_{\mu}^{0}=(A,0,0,0), e_{\mu}^{1}=(0,B,0,0), e_{\mu}^{2}=(0,0,rC,0), e_{\mu}^{3}=(0,0,0,rC\sin\theta), \quad (3.1)$$

and

$$A_{\mu}^{01}=(U,0,0,0), A_{\mu}^{12}=(0,0,W,0), A_{\mu}^{13}=(0,0,0,Z\sin\theta), \quad (3.2a)$$

$$A_{\mu}^{23}=(0,0,0,V\cos\theta), A_{\mu}^{02}=\omega_{\mu}^{03}=0,3. \quad (3.2b)$$

where A, B, C, U, V, Z and W are functions only of the three-dimensional radius r. In addition, the spherically symmetric $SU(2)$ gauge fields will be parametrized as (Witten ansatz):

$$A=uT_3dt+w(T_2d\theta-T_1\sin\theta d\varphi)+T_3\cos\theta d\varphi, \quad (3.3)$$

where u and w are functions also depending only on r.

We use the above expressions to compute the components of the tensors $F_{\mu\nu}^{i}$ and $F_{\mu\nu}^{ij}$. Their's non-null components are:

$$F_{10}^{0}=A'+UB, \quad F_{12}^{2}=C+rC'-WB, \quad (3.3a)$$

$$F_{13}^{3}=(C+rC'-ZB)\sin\theta, \quad F_{23}^{3}=rC\cos\theta(1-V), \quad (3.3b)$$

and respectively:

$$F_{10}^{01}=U'+4\lambda^2AB, F_{20}^{02}=(UW+4\lambda^2rAC), \quad (3.4a)$$

$$F_{30}^{03}=\sin\theta(UZ+4\lambda^2rAC), \quad F_{21}^{21}=W'-4\lambda^2rBC, \quad (3.4b)$$

$$F_{31}^{31}=(Z'-4\lambda^2rBC)\sin\theta, \quad F_{31}^{32}=V'\cos\theta, \quad (3.4c)$$

$$F_{32}^{31}=(Z-VW)\cos\theta, F_{32}^{23}=(V-ZW+4\lambda^2r^2C^2)\sin\theta, \quad (3.4d)$$

where A', C', U', V', W', and Z' denotes the derivatives with respect to the variable r. Analogously, we obtain the following non-null components of the $SU(2)$ stress tensor $G_{\mu\nu}^{\alpha}$:

$$G_{02}^{1}=-uw, G_{13}^{1}=-w'\sin\theta, G_{03}^{2}=-uw\sin\theta, \quad (3.5a)$$

$$G_{12}^{2}=-w', G_{01}^{3}=-u', G_{23}^{3}=(w^2-1)\sin\theta, \quad (3.5b)$$

with $u'=\dfrac{du}{dr}$ and $w'=\dfrac{dw}{dr}$.

The integral action of our model is:

$$S_{EYM}=\int d^4xe\left\{-\frac{1}{16\pi G}(F-2\Lambda)-\frac{1}{4Kg^2}Tr(G_{\mu\nu}G^{\mu\nu})\right\}, \quad (3.6)$$

where $F = F^{ij}_{\mu\nu}\bar{e}^{\mu}_{i}\bar{e}^{\nu}_{j}$, $e = \det\left(e^{i}_{\mu}\right)$ and $G_{\mu\nu} = G^{\alpha}_{\mu\nu}T_{\alpha}$, $G^{\mu\nu} = G^{\beta\mu\nu}T_{\beta}$. We choose $Tr\left(T_{\alpha}T_{\beta}\right) = K\delta_{\alpha\beta}$; for $SU(2)$ group we have $T_{a} = \frac{1}{2}\tau_{a}$ (τ_{a} being the Pauli matrices) and then $K = \frac{1}{2}$. The gravitational constant G is the only dimensional quantity in action (the units $\hbar = c = 1$ are understood) and g is the $SU(2)$ coupling constant. Taking $\delta S_{EYM} = 0$ with respect to A^{α}_{μ}, e^{i}_{μ} and A^{ij}_{μ}, we obtain respectively the following field equations [9]:

$$\frac{1}{e}\partial_{\mu}\left(eG^{\alpha\mu\nu}\right) + \varepsilon^{\alpha\beta\gamma}A^{\beta}_{\mu}G^{\gamma\mu\nu} = 0, \tag{3.7}$$

$$F^{i}_{\mu} - \frac{1}{2}\left(F - 2\Lambda\right)e^{i}_{\mu} = 8\pi G T^{i}_{\mu}, \tag{3.8}$$

where T^{i}_{μ} is the energy-momentum tensor of the $SU(2)$ gauge fields

$$T^{i}_{\mu} = \frac{1}{Kg^{2}}\left(G^{\alpha}_{\mu\nu}G^{\nu i}_{\alpha} - \frac{1}{4}e^{i}_{\mu}G^{\alpha}_{\rho\lambda}G^{\rho\lambda}_{\alpha}\right), \tag{3.9}$$

and

$$F^{i}_{\mu\nu} = 0. \tag{3.10}$$

In Eq. (3.9) we denoted $G^{\nu i}_{\alpha} = \eta^{ij}\,\bar{e}^{\rho}_{j}G^{\nu}_{\alpha\rho}$. The Eq. (3.10) is equivalent with the vanishing of the torsion in a Riemann-Cartan theory and determine the gauge potentials A^{ij}_{μ} as function of tetrad fields e^{ν}_{μ}. Then, introducing (3.4) and (3.5) into these field equations and imposing the constraints $C = 1$, $A = \frac{1}{B} = \sqrt{N}$ with $N(r)$ a new unknown positive defined function, we obtain:

$$\left(Nw'\right)' = \frac{w\left(w^{2}-1\right)}{r^{2}} - \frac{u^{2}w}{N}, \tag{3.11a}$$

$$\left(r^{2}u'\right)' = \frac{2uw^{2}}{N}, \tag{3.11b}$$

$$\frac{w'^{2}}{r} + \frac{u^{2}w^{2}}{rN^{2}} = 0, \tag{3.11c}$$

$$\frac{1}{2}\left(rN' + N - 1\right) + \frac{r^{2}u'^{2}}{2} + \frac{u^{2}w^{2}}{N} + Nw'^{2} + \frac{\left(w^{2}-1\right)^{2}}{2r^{2}} + \frac{2\Lambda r^{2}}{3} = 0, \tag{3.11d}$$

where we used $K = \frac{1}{2}$ and $\frac{4\pi G}{g^{2}} = 1$ units. These equations admit the following solution of Schwarzschild-Reissner-Nordstrom-de-Sitter type with a nontrivial gauge field describing colored black holes [15]:

$$u(r) = u_0 + \frac{Q}{r}, \quad w(r) = 0, \quad N(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2, \tag{3.12a}$$

where $\Lambda = -12\lambda^2$ is the cosmological constant of the model. They admit also the self-dual solution (Schwarzschild):

$$u(r) = 0, \quad w(r) = \pm 1, \quad N(r) = 1 - \frac{2m}{r}. \tag{3.12b}$$

But, the solution (3.12a) is not a self-dual one.

4. NONCOMMUTATIVITY CORRECTIONS

We suppose now that the space-time is noncommutative, *i.e.* its coordinates $x^\mu = (r, \theta, \varphi, t)$ satisfy the (canonical) commutation relations [16]:

$$\left[x^\mu, x^\nu\right] = i\,\Theta^{\mu\nu}, \tag{4.1}$$

where $\Theta^{\mu\nu} = -\Theta^{\nu\mu}$ are constant parameters. It is known that the noncommutativity field theory on such a space-time requires the introduction of the star "*" product between the fields defined on this space-time:

$$(\Phi * \Psi)(x) = \Phi(x)\, e^{\frac{i}{2}\Theta^{\mu\nu}\overleftarrow{\partial}_\mu \otimes \overrightarrow{\partial}_\nu}\, \Psi(x). \tag{4.2}$$

In order to calculate the effect of the noncommutativity on the gauge fields we use the Seiberg-Witten map [17]. This map gives the deformed (noncommutative) gauge fields as a series of parameter $\Theta^{\mu\nu}$, containing the commutative gauge fields and their derivatives. For simplicity we will consider only the space-space commutatitvity and choose:

$$\Theta^{\mu\nu} = \begin{pmatrix} 0 & \Theta & 0 & 0 \\ -\Theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.3}$$

where Θ is a constant parameter of deformation. Proceeding along the approach of [18], we will obtain a deformed Reissner-Nordström-de-Sitter solution in noncommutative gauge theory of gravitation.

In our case of $SU(2)\times SO(4,1)$ gauge symmetry, the noncommutative tetrad fields $\hat{e}^i_\mu$ up to the second order in parameter Θ are given by

$$\hat{e}^1_1 = \frac{1}{A} + \frac{1}{8}\left(A'' - \frac{\Lambda r A'}{12A^2} - \frac{5\Lambda}{12A} + \frac{5\Lambda^2 r^4}{3}\right)\Theta^2 + O\left(\Theta^3\right),$$

$$e_1^2 = -i\frac{\Lambda r}{12A^2}\Theta + O\left(\Theta^3\right)$$

$$\hat{e}_2^1 = -\frac{i}{4}\left(A + 2rA' - \frac{\Lambda r^2}{3A}\right)\Theta + O\left(\Theta^3\right),$$

$$\hat{e}_2^2 = r + \frac{1}{32}(7AA' + 12rA'^2 + 12rAA' - \frac{11\Lambda r}{3} - \tag{4.4}$$

$$-\frac{5\Lambda r^2 AA'}{3} + \frac{5\Lambda^2 r^3}{9})\,\Theta^2 + O\left(\Theta^3\right),$$

$$\hat{e}_3^3 = r\sin\theta - \frac{i}{4}\left(\cos\theta\right)\Theta + \frac{1}{8}(2rA'^2 + rAA' + 2AA' - \frac{A'}{A}$$

$$+\frac{\Lambda r}{3A^2} - \frac{4\Lambda r^2 A'}{3A} + \frac{2\Lambda^2 r^3}{A^2} - \frac{11\Lambda r}{12})\left(\sin\theta\right)\Theta^2 + O\left(\Theta^3\right),$$

$$\hat{e}_0^0 = A + \frac{1}{8}(2rA'^3 + 5rAA'A'' + rA^2A''' + 2AA'^2 + A^2A'' -$$

$$-\Lambda rA' + \frac{\Lambda^2 r^2}{6A} - \frac{\Lambda r^2 A'^2}{3A} - \frac{\Lambda r^2 A''}{3} - \frac{\Lambda A}{4})\,\Theta^2 + O\left(\Theta^3\right),$$

where A', A'', A''' are the first, second and third derivatives of $A(r)$, respectively, with $A = \sqrt{N}$ and N given by (3.12a).

Using the hermitian conjugate $\hat{e}_\mu^{i+}\left(x,\Theta\right)$ of the deformed tetrad fields given in (4.4), we can define a real deformed metric by formula [19]:

$$\hat{g}_{\mu\nu}\left(x,\Theta\right) = \frac{1}{2}\eta_{ij}\left(\hat{e}_\mu^i * \hat{e}_\nu^{j+} + \hat{e}_\mu^j * \hat{e}_\nu^{i+}\right), \tag{4.5}$$

where $\eta_{ij} = diag(1, 1, 1, -1)$, i, j = 1, 2, 3, 0. The non-null components of this metric are:

$$\hat{g}_{11} = \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1} +$$

$$+\frac{(-2mr^3 + 3m^2r^2 + 3Q^2r^2 - 6mQ^2r + 2Q^4)}{16r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)}\Theta^2 +$$

$$+\frac{\left(\frac{\Lambda^2 r^8}{3} - \frac{3\Lambda r^6}{4} + \frac{11mr^5}{4} - \frac{7\Lambda Q^2 r^4}{3}\right)}{16r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)}\Theta^2 + O\left(\Theta^4\right),$$

$$\hat{g}_{22}\left(r,\Theta\right) = r^2 + \frac{(r^4 - 17mr^3 - 34m^2r^2 + 27Q^2r^2 - 75mQ^2r + 30Q^4)}{4r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)^2}\Theta^2 + \tag{4.6}$$

$$+\frac{\left(-\frac{56\Lambda^2 r^8}{3}+\frac{38\Lambda r^6}{3}-24m\Lambda r^5+\frac{46\Lambda Q^2 r^4}{3}\right)}{4r^2(r^2-2mr+Q^2-\frac{\Lambda}{3}r^4)^2}\Theta^2+O\left(\Theta^4\right),$$

$$\hat{g}_{33}=r^2\sin\theta+\frac{\cos^2\theta\,(r^4+2mr^3-7Q^2r^2-4m^2r^2+16mQ^2r-8Q^4)}{16r^2(r^2-2mr+Q^2-\frac{\Lambda}{3}r^4)}\Theta^2+$$

$$+\frac{(-mr^3+m^2r^2+2Q^2r^2-4mQ^2r+2Q^4)}{14r^2(r^2-2mr+Q^2-\frac{\Lambda}{3}r^4)}\Theta^2$$

$$+\frac{\sin^2\theta\left(-\frac{14m\Lambda r^3}{3}+\Lambda Q^2r^2-\frac{25\Lambda^2r^6}{9}+\frac{7\Lambda r^4}{3}\right)}{16r^2(r^2-2mr+Q^2-\frac{\Lambda}{3}r^4)}\Theta^2+O(\Theta^4),$$

$$\hat{g}_{00}=-\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{\Lambda}{3}r^2\right)-\frac{4mr^3-9Q^2r^2-11m^2r^2+30mQ^2r-14Q^4}{4r^6}\Theta^2+$$

$$+\frac{\Lambda\left(6mr+25\Lambda r^4-9r^2-9Q^2\right)}{144r^2}\Theta^2+O\left(\Theta^4\right).$$

It is important to remark that for $\Lambda\to 0$ we obtain from (4.6) the noncommutativity corrections for the Reisner-Nordström metric, and for $\Lambda\to 0$ and simultaneously $Q\to 0$, we obtain the corrections to the Schwarzschild metric [19].

If we consider that the source of the gravitational field is a black hole, then we can calculate the noncommutativity corrections to the horizon radius, temperature and entropy [20].

The deformed $SU(2)$ gauge fields up to the first order in $\Theta^{\mu\nu}$ are given by [21]:

$$\hat{A}_\sigma=A_\sigma-\frac{1}{4}\Theta^{\mu\nu}\left(\left\{A_\mu,\partial_\nu A_\sigma\right\}-\left\{G_{\mu\sigma},A_\nu\right\}\right). \tag{4.7}$$

In this case we have to use the enveloping algebra of $SU(2)$ which coincides with the Lie algebra of $U(2)$ group. Then the gauge potentials are $A_\mu=\left(B_\mu,A_\mu^\alpha\right)$, where B_μ is a new gauge fields introduced by enveloping algebra. For simplicity, we chose $B_\mu=(1/2,0,0,0)$ as a constant field.

Introducing (3.3) in (4.7), we obtain the following corrections for $SU(2)$ gauge fields, up to the first order in parameter Θ:

$$\hat{A}_3^1=-w\sin\theta+\frac{1}{4}u\,w\,\Theta+O\left(\Theta^2\right),$$

$$\hat{A}_0^1 = -\frac{1}{4} u\, w\, \Theta + O(\Theta^2),$$

$$\hat{A}_1^2 = \frac{1}{2} w'\, \Theta + O(\Theta^2),$$

$$\hat{A}_2^2 = w + O(\Theta^2), \tag{4.8}$$

$$\hat{A}_3^3 = \cos\theta + \frac{1}{4}\left(2 - w^2\right)\sin\theta\, \Theta + O(\Theta^2),$$

$$\hat{A}_0^3 = u + O(\Theta^2).$$

In particular, if we use the solution (3.12a), then we have only two non-null components:

$$\hat{A}_3^3 = \cos\theta + \frac{1}{2}\sin\theta\, \Theta + O(\Theta^2), \tag{4.9}$$

$$\hat{A}_0^3 = u_0 + \frac{Q}{r} + O(\Theta^2).$$

More general case of $SU(n)$ group can be studied in a similar way.

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