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Gauge theories on noncommutative space-time

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Abstract

We study the gauge theories with de-Sitter and Poincaré groups as local symmetries defined on a noncommutative space-time. First of all we discuss the twist as a symmetry principle for noncommutatve gauge theories. It is shown that external (Poincaré or de-Sitter) symmetry and internal gauge symmetry cannot be unified under a common twist. Then, the de-Sitter gauge theory on commutative space-time is formulated considering the case of spherical symmetry. The Poincaré gauge theory is obtained in the limit of vanishing cosmological constant $\Lambda \to 0$. The noncommutative gauge theory is developed using the Seiberg-Witten map in order to establish a connection with the usual (commutative) case. We obtain noncommutativity corrections for the Schwarzschild and Reissner-Nordström-de-Sitter metrics. Some applications to the study of the thermodynamic properties of the black holes are also analyzed.

1 Introduction

The interest in noncommutative space-time is motivated specially by the hope of obtaining a quantum theory of gravitation [1, 9]. The most frequent utilized (canonical) condition of noncommutativity for the space-time coordinates is:

$$[x^\mu,\ x^\nu] = i\ \Theta^{\mu\nu}, \tag{1.1}$$

where $\Theta^{\mu\nu}$ is a constant antisymmetric matrix. This condition is similar with the Heisenberg relation $[p_j,\ q_k] = \frac{\hbar}{i}\delta_{jk}$ in quantum mechanics. The noncommutativity condition (1.1) introduces an universal minimal length scale. At distances near $\sqrt{\Theta}$ the classical concept of smooth space-time manifold breaks down [21]. It is generally assumed that $\sqrt{\Theta}$ is closed to the Planck length $L_p = 1.6\cdot 10^{-35}\ m$. Therefore, we can have a quantization of the space-time and this explains the interest in noncommutativity [19, 20, 21, 22, 23]. One important problem consists in formulating gauge theories, both internal and external, compatible with the noncommutativity propriety of the space-time. We will present some results on this subject, emphasizing some open questions regarding noncommutative gauge theories.

Considering first the commutative (usual) gauge theory, let us suppose that we have the Lagrangian for a Dirac field $\Psi(x)$:

$$L_D = \bar{\Psi}\left(i\gamma^\mu\partial_\mu - m\right)\Psi. \tag{1.2}$$

Then, we consider a Lie group G and suppose that the Lagrangian L_D invariant under its transformations. A transformation of G can be written as:

$$U = e^{i\alpha^a T_a},\ a = 1, 2, ..., N, \tag{1.3}$$

where α^a are constant parameters of the Lie group and T_a - its infinitesimal generators with the property:

$$[T_a,\ T_b] = f^c_{ab}T_c. \tag{1.4}$$

Here, $f^c_{ab} = -f^c_{ba}$ are the constants of structure of the symmetry group G. We say that G is a global Lie group of symmetry.

Now, if we suppose that the group parameters α^a depend on space-time coordinates, $\alpha^a = \alpha^a(x)$, then we say that we have a gauge (local) symmetry. In order to assure also the invariance of L_D under the gauge transformations, we have to introduce new gauge fields $A_\mu^a(x)$, and define a gauge covariant derivative

$$D_\mu = \partial_\mu - gA_\mu, \quad A_\mu = A_\mu^a T_\mu. \tag{1.5}$$

where g is the gauge coupling constant. Then we define the strength tensor associated to the gauge fields through the relation

$$gF_{\mu\nu} = [D_\mu,\ D_\nu] = g\left(\partial_\mu A_\nu - \partial_\nu A_\mu + g\left[A_\mu,\ A_\nu\right]\right), \tag{1.6}$$

or, by components

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{bc}^a A_\mu^b A_\nu^c. \tag{1.7}$$

The Lagrangian of the gauge fields $A_\mu^a(x)$ above introduced is chosen as

$$L_g = -\frac{1}{2}tr\left(F_{\mu\nu}F^{\mu\nu}\right), \tag{1.8}$$

where we understand the condition $tr\left(T_aT_b\right) = \frac{1}{2}\delta_{ab}$ which is valid for any compact Lie group.

The gauge invariant Lagrangian of the system (Dirac and gauge fields) is

$$L = L_D + Lg = \bar{\Psi}\left(i\gamma^\mu D_\mu - m\right)\Psi - \frac{1}{2}tr\left(F_{\mu\nu}F^{\mu\nu}\right). \tag{1.9}$$

We can see that the usual derivative is changed by the gauge covariant derivative:

$$\partial_\mu \to D_\mu \tag{1.10}$$

in order to assure the gauge invariance of the Lagrangian L.

Coming back to the noncommutative theory (NC), we remember that the coordinates on the space-time manifold M do not commute [see condition (1.1)], meaning that the algebra A of the functions defined over M is deformed to an associative but noncommutative new algebra A_Θ. The problem with symmetries (global and local) in NC models is that the matrix $\Theta^{\mu\nu}$ is a parameter rather than a dynamical variable. Therefore, $\Theta^{\mu\nu}$ must not be transformed under the symmetry group action. This inconsistency makes impossible to preserve usual Poincaré and diffeomorphism invariances in NC theories.

Let us take, for example, two infinitesimal gauge transformations with the parameters $\alpha(x) = \alpha^a(x)\, T_a$, $\beta(x) = \beta^b(x)\, T_b$. Their commutator (in the commutative case)

$$\left[\alpha(x),\ \beta(x)\right] = \alpha^a(x)\,\beta^b(x)\left[T_a,\ T_b\right], \tag{1.11}$$

is again a gauge transformation. In the NC case, one introduces a star-product ($\star$ - product) on the algebra A_Θ of functions over the space-time (noncommutative)

$$\left(f \star g\right)(x) = f(x)\ e^{-\frac{i}{2}\Theta^{\mu\nu}\overleftarrow{\partial}_\mu \otimes \overrightarrow{\partial}_\nu} g(x) \tag{1.12}$$

Then, a natural generalization of the gauge transformations for a field $\Phi(x)$ is:

$$\delta_\alpha \Phi = \alpha(x) \star \Phi(x)$$

The commutator of the two transformations reads:

$$\left[\alpha(x),\ \beta(x)\right]_\star = \alpha(x) \star \beta(x) - \beta(x) \star \alpha(x) =$$

$$= \alpha^a(x) \star \beta^b(x)\, T_aT_b - \beta^b(x) \star \alpha^a(x)\, T_bT_a$$

or

$$\left[\alpha(x),\ \beta(x)\right]_\star = \frac{1}{2}\left(\alpha^a(x) \star \beta^b(x) + \beta^b(x) \star \alpha^a(x)\right)\left[T_a,\ T_b\right] -$$

$$-\frac{1}{2}\left(\alpha^a(x)\star\beta^b(x)-\beta^b(x)\star\alpha^a(x)\right)\{T_a,\ T_b\}$$

Therefore, the set of generators T_a must be closed with respect to both commutators and anti-commutators. As a consequence, only unitary groups $U(n)$ can be used as symmetries, or we have to work in the enveloping algebra of the Lie group of symmetry which can be organized as a Hopf algebra H (doted with unit 1, a co-product Δ and a co-unit ε). In the case of unitary Lie groups $U(n)$, the Hopf algebra coincides with the Lie algebra itself.

An important operator on a Hopf algebra is the twist F. It is defined as an element $F \in H \otimes H$ that is invertible and satisfies the properties

$$(F\otimes 1)(\Delta\otimes id)F=(1\otimes F)(id\otimes\Delta)F \tag{1.13}$$

$$(id\otimes\varepsilon)\Delta=id=(\varepsilon\otimes id)\Delta \tag{1.14}$$

As an example of twist, we give the following one which is named also abelian twist:

$$F=e^{-\frac{i}{2}\Theta^{\mu\nu}\partial_\mu\otimes\partial_\nu} \tag{1.15}$$

and it will be used in what follows.

Another problem in NC theories is connected with the gauge covariant derivative D_μ. In order to obtain a gauge invariant twist element we have to change the partial derivatives ∂_μ by the gauge covariant derivatives D_μ. Then, we can define the non-abelian covariant twist [18]:

$$T=e^{-\frac{i}{2}\Theta^{\mu\nu}D_\mu\otimes D_\nu} \tag{1.16}$$

Then, we can use this twist element to define the multiplication map,

$$(f\star_T g)(x)=\mu\circ T^{\ -1}(f\otimes g)(x)=\mu_{\star_T}^{-1}(f\otimes g)(x)\,. \tag{1.17}$$

where $\mu(f\otimes g)=fg$ is the usual (commutative) multiplication map. The algebra $A_{\star_T}$ equipped with this new $\star_T$ - product is non-commutative and non- associative.

The Leibniz rule for a gauge transformation (1.17) is not satisfied now:

$$\delta_\alpha(\Phi\otimes\Psi)\neq\delta_\alpha\Phi\otimes\Psi+\Phi\otimes\delta_\alpha\Psi \tag{1.18}$$

This introduces many difficulties in a field theory on the NC space-time.

2 Twist as a symmetry principle

Recently, an attempt was made to twist also the gauge algebra, i.e. to extend the Poincaré algebra by the gauge algebra, as semidirect product, and to twist the coproduct of the gauge generators with the same Abelian twist [13, 29, 30]. The result seemed to be spectacular: the same theories, which previously were shown to be subject to the no-go theorem, were now claimed to be invariant under any usual (not noncommutative) gauge group and to admit any representations, just as in the commutative case. The latter approach was shown however to be in conflict with the very idea of gauge transformations, since it assumed implicitly that if a field is transformed according to a given representation of the gauge algebra, then its derivatives of any order also transform according to the representations of the gauge algebra, which is obviously not the case. A new approach to noncommutative gauge theories seems to be necessary and this necessity arises in particular from the gravitational theory: NC gravitational effects have been recently calculated from string theory with antisymmetric background field, i.e. in the same theory as the one which gave rise, in the low-energy limit, to the usual noncommutative field theories. It turns out that, in the case of NC gravitational interactions, string theory contains a much richer dynamics than the one of the theories constructed in terms of Moyal -product alone.

The question arises whether the concept of twist appears as a symmetry principle in constructing NC field theories: any symmetry that such theories may enjoy, be it space-time or internal symmetry, global or local, should be formulated as a twisted symmetry. In pursuit of this idea, in this letter we take the most general ansatz for a non-Abelian twist, which, in the absence of the gauge interaction, reduces to the Abelian twist. We shall show that the twisting of the gauge transformations is not possible, in a manner compatible with the representations of the gauge algebra and keeping at the same time the Moyal space defined by as underlying space of the theory.

In [17] it was shown in detail that the use of the Abelian twist (1.13) for deforming the Hopf algebra is not compatible with the concept of gauge transformations. We recall that the reason for this conflict is the fact that the derivatives of a field do not transform according to the representations of the gauge algebra, as the fields themselves do.

However, the covariant derivatives of a field transform exactly according to the same representation as the field itself, as we have mentioned above. Thus the option of defining the non-Abelian twist element (1.16) involving covariant derivatives naturally occurs [18]. The operator T defined in (1.16) satisfies the property (1.14) but do not verifies (1.13) which assures the associativity of the product $\star_T$-product. The second order terms which do not cancel in (1.13) are, in the l.h.s.

$$T_1 = \frac{1}{2}\left(-\frac{i}{2}\right)^2 \Theta^{\mu\nu}\Theta^{\rho\sigma}(D_\rho \otimes D_\mu \otimes D_\sigma D_\nu + D_\mu \otimes D_\rho \otimes D_\sigma D_\nu + \\ +2D_\mu D_\rho \otimes D_\nu \otimes D_\sigma + 2D_\mu \otimes D_\nu D_\rho \otimes D_\sigma) \tag{2.1}$$

and in r.h.s.

$$T_2 = \frac{1}{2}\left(-\frac{i}{2}\right)^2 \Theta^{\mu\nu}\Theta^{\rho\sigma}(2D_\rho \otimes D_\mu \otimes D_\sigma D_\nu + D_\rho D_\mu \otimes D_\sigma \otimes D_\nu + \\ D_\rho D_\mu \otimes D_\nu \otimes D_\sigma + 2D_\rho \otimes D_\mu D_\sigma \otimes D_\nu) \tag{2.2}$$

We can try to obtain their canceling by adding supplementary terms of second order in the exponent of (1.16). The most general form with meaningful terms of second order in , which would satisfy (1.13), is:

$$T = \exp\{-\frac{i}{2}\Theta^{\mu\nu} D_\mu \otimes D_\nu + \frac{1}{2}\left(-\frac{i}{2}\right)^2 \Theta^{\mu\nu}\Theta^{\rho\sigma}[aD_\mu \otimes D_\sigma D_\nu D_\rho + \\ +bD_\mu \otimes D_\nu D_\sigma D_\rho + cD_\mu \otimes D_\sigma D_\rho D_\nu + a' D_\sigma D_\nu D_\rho \otimes D_\mu + \\ +b' D_\nu D_\sigma D_\rho \otimes D_\mu + c' D_\sigma D_\rho D_\nu \otimes D_\mu + O\left(\Theta^2\right)]\} \tag{2.3}$$

where $a, b, c, a\prime, b\prime, c\prime$ are constants which have to be determined by imposing (1.13) up to the second order in Θ. By a direct calculation, it can be verified that the twist condition can not be satisfied, and consequently there are no second order terms, formulated in terms of covariant derivatives, which can lead to the fulfillment of the twist condition (1.13) up to the second order in Θ. It can be also verified that, by relaxing the requirement of exponential form for the twist as in (1.16) to an arbitrary invertible function $F(X)$, i.e. by taking the first and second derivatives $F\prime(0)$ and F"(0) (the coefficients of the -expansion of the twist) to be arbitrary, the twist condition (1.13) still cannot be satisfied. Thus the result is general and is not based on the requirement of "correspondence principle".

Having in view also the analysis of [17], which showed that the Abelian twist (1.15) cannot be used for twisting gauge transformations, it appears that there is no way to reconcile the twist condition and the gauge invariance principle. Let us mention that by using the Seiberg-Witten map [2], which provides a connection between a NC gauge symmetry and the corresponding commutative one as a power series in the noncommutativity parameter Θ, the resulting Lagrangian or action [31] cannot be brought to the form given by a twist.

It is intriguing that the external Poincaré symmetry and the internal gauge symmetry cannot be unified under a common twist. The situation is reminiscent of the Coleman-Mandula theorem [32] (for a pedagogical presentation and other references, see [33]), although not entirely, since the Coleman-Mandula theorem concerns global symmetry and simple groups. However, one can envisage that supersymmetry [34], due to its intrinsic internal symmetry, may reverse the situation, and a noncommutative supersymmetric gauge theory can be constructed by means of a twist.

3 Commutative gauge theory

We consider a model of gauge theory for gravitation having the de-Sitter group (DS) as local symmetry. The base manifold is a four-dimensional Minkowski space-time M_4, in spherical coordinates:

$$ds^2 = dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{3.1}$$

The corresponding metric $g_{\mu\nu}$ has the following non-null components:

$$g_{00} = 1, \qquad g_{11} = -1, \qquad g_{22} = -r^2, \qquad g_{33} = -r^2\sin^2\theta. \tag{3.2}$$

The gravitational field will be described by gauge potentials and the mathematical structure of this underlying space-time is not affected by physical events [3].

The group DS is 10 dimensional, and its infinitesimal generators will be denoted by Π_a and $M_{ab} = -M_{ba}$, $a, b = 0, 1, 2, 3$, where Π_a generate the de-Sitter "translations" and M_{ab} - the Lorentz transformations [6]. In order to give a general formulation of the gauge theory for the de-Sitter group DS, we will denote the generators Π_a and M_{ab} by X_A, $A = 1, 2, \ldots, 10$. Then, the equations of structure can be written in the general form:

$$[X_A, X_B] = i f_{AB}^{C} X_C, \tag{3.3}$$

where $f_{AB}^{C} = -f_{BA}^{C}$ are the constants of structures whose expressions will be given below [see Eqs. (3.7)].

Let us suppose now that DS is a gauge group for gravitation. As usual, we introduce 10 gauge fields (the gravitational potentials) $h_\mu^A(x)$, $A = 1, 2, ..., 10$, $\mu = 0, 1, 2, 3$, where $(x) = (x^\mu)$ denotes the local coordinates on M_4, with $x^0 = t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$. Then we construct the 2-form $\mathcal{F}$ associated to the potentials $h_\mu^A(x)$ [5]:

$$\mathcal{F} = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu, \tag{3.4}$$

where $F_{\mu\nu} = F_{\mu\nu}^A X_A$ are its components which take values in Lie algebra of DS group (Lie-algebra valued). The components $F_{\mu\nu}^A$ are given by:

$$F_{\mu\nu}^A = \partial_\mu h_\nu^A - \partial_\nu h_\mu^A + f_{BC}^A h_\mu^B h_\nu^C. \tag{3.5}$$

In order to write the constant of structure f_{AB}^C in a compact form, we use the following notations for the index A:

$$A = \begin{cases} a = 0, 1, 2, 3, \\ [ab] = [01], [02], [03], [12], [13], [23]. \end{cases} \tag{3.6}$$

This means that A can stand for a single index like 2 or 3, as well as for a pair of indices like [01], [12] etc. The infinitesimal generators X_A of the DS group are interpreted as: $X_a \equiv \Pi_a$ (the de-Sitter "translation" operators) and $X_{[ab]} \equiv M_{ab}$ (the Lorentz transformation operators) with the property $M_{ab} = -M_{ba}$. The constants of structures f_{AB}^C have then the following expressions:

$$\begin{aligned} f_{bc}^a &= f_{c[de]}^{[ab]} = f_{[bc][de]}^a = 0, \\ f_{cd}^{[ab]} &= 4\lambda^2 \left(\delta_c^b \delta_d^a - \delta_c^a \delta_d^b\right) = -f_{dc}^{[ab]}, \\ f_{b[cd]}^a &= -f_{[cd]b}^a = \frac{1}{2}\left(\eta_{bc}\delta_d^a - \eta_{bd}\delta_c^a\right), \\ f_{[ab][cd]}^{[ef]} &= \frac{1}{4}\left(\eta_{bc}\delta_a^e\delta_d^f - \eta_{ac}\delta_b^e\delta_d^f + \eta_{ad}\delta_b^e\delta_c^f - \eta_{bd}\delta_a^e\delta_c^f\right) - e \longleftrightarrow f, \end{aligned} \tag{3.7}$$

where λ is a real parameter, and (η_{ab}) =diag$(1, -1, -1, -1)$ is the Minkowski metric in the tangent space to M_4. In fact, the constants (3.7) correspond to a deformation of de-Sitter Lie algebra having λ

as parameter [6]. If we consider the contraction $\lambda \longrightarrow 0$ then the generators Π_a become the generators P_a of space-time translations and the group DS contracts therefore to the Poincaré group P.

We denote the gravitational gauge fields $h_\mu^A(x)$ by $e_\mu^a(x)$, if $A = a$, and respectively $\omega_\mu^{ab}(x) = -\omega_\mu^{ba}(x)$ if $A = [ab]$. Then, the components $F_{\mu\nu}^A$ of the strength tensor can be written in the form [7]:

$$F_{\mu\nu}^a = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a + \left(\omega_\mu^{ab} e_\nu^c - \omega_\nu^{ab} e_\mu^c\right) \eta_{bc} \tag{3.8}$$

$$F_{\mu\nu}^{ab} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \left(\omega_\mu^{ac} \omega_\nu^{db} - \omega_\nu^{ac} \omega_\mu^{db}\right) \eta_{cd} + 4\lambda^2 \left(\delta_c^b \delta_d^a - \delta_c^a \delta_d^b\right) e_\mu^c e_\nu^d. \tag{3.9}$$

The integral of action associated to the gravitational gauge fields $e_\mu^a(x)$ and $\omega_\mu^{ab}(x)$ will be chosen as [7]:

$$S_g = \frac{1}{16\pi G} \int d^4x \, e \, F, \tag{3.10}$$

where $e = \det(e_\mu^a)$ and

$$F = F_{\mu\nu}^{ab} \, \bar{e}_a^\mu \, \bar{e}_b^\nu. \tag{3.11}$$

Here, $\bar{e}_a^\mu(x)$ denotes the inverse of $e_\mu^a(x)$ satisfying the usual properties [7]:

$$e_\mu^a \bar{e}_b^\mu = \delta_b^a, \qquad e_\mu^a \bar{e}_a^\nu = \delta_\mu^\nu. \tag{3.12}$$

We suppose that the source of the gravitation creates also an electromagnetic field $A_\mu(x)$. The corresponding integral of the action will be chosen in the form [7]:

$$S_{em} = -\frac{1}{4Kg^2} \int d^4x \, e \, A_\mu^a \, \overline{A}_a^\mu, \tag{3.13}$$

with A_μ^a and $\overline{A}_a^\mu$ defined as:

$$A_\mu^a = A_\mu^\nu e_\nu^a, \qquad A_\mu^\nu = \bar{e}_a^\nu \bar{e}_b^\rho \eta^{ab} A_{\mu\rho}, \tag{3.14}$$

and respectively:

$$\overline{A}_a^\mu = A_\mu^\nu \bar{e}_a^\nu. \tag{3.15}$$

Here K is a constant that will be chosen in a convenient form to simplify the solutions of the field equations and $A_{\mu\rho}$ denotes the electromagnetic field tensor:

$$A_{\mu\rho} = \partial_\mu A_\rho - \partial_\rho A_\mu. \tag{3.16}$$

The quantity g in (3.13)is the gauge coupling constant [7]. Then, the total integral of action associated to the system composed of the two fields is given by the sum of the expressions (3.10) and (3.13):

$$S = \int d^4x \left[\frac{1}{16\pi G} F - \frac{1}{4Kg^2} A_\mu^a \, \overline{A}_a^\mu\right] e. \tag{3.17}$$

The field equations for the gravitational potentials $e_\mu^a(x)$ can be obtained by imposing the variational principle $\delta_e S = 0$ with respect to $e_\mu^a(x)$. They are [18, 35]:

$$F_\mu^a - \frac{1}{2} F \, e_\mu^a = 8\pi G T_\mu^a, \tag{3.18}$$

where F_μ^a is defined by:

$$F_\mu^a = F_{\mu\nu}^{ab} \, \bar{e}_b^\nu, \tag{3.19}$$

and T_μ^a is the energy-momentum tensor of the electromagnetic field [6, 7]:

$$T_\mu^a = \frac{1}{Kg^2} \left(A_\mu^b \, A_\nu^a \, \bar{e}_b^\nu - \frac{1}{4} A_\nu^b \, A_b^\nu \, e_\mu^a\right). \tag{3.20}$$

The field equations for the other gravitational gauge potentials $\omega_\mu^{ab}(x)$ are equivalent with [6]:

$$F_{\mu\nu}^a = 0. \tag{3.21}$$

We will obtain a solution of the field equations (3.18) and (3.21) supposing that the gravitational field has spherical symmetry and it is created by a point-like mass m (the source). In the same time, we will admit that the electromagnetic field $A_\mu(x)$ is due to a constant electric charge Q of the same source, i.e. we will consider that the point-like mass m is also electrical charged.

The field equations for the electromagnetic field $A_\mu(x)$ can be obtained also by imposing the variational principle $\delta_A S = 0$. However, we will not write these equations because the expressions of the $A_\mu(x)$ components are well defined for a fixed point-like particle with charge Q.

We consider now a particular form of spherically gravitational gauge field given by the following ansatz [7]:

$$e_\mu^0 = (A, 0, 0, 0)\,, \quad e_\mu^1 = \left(0, \frac{1}{A}, 0, 0\right), \quad e_\mu^2 = (0, 0, r, 0)\,, \quad e_\mu^3 = (0, 0, 0, r\sin\theta)\,, \tag{3.22}$$

and

$$\begin{gathered}\omega_\mu^{01} = (U, 0, 0, 0)\,, \quad \omega_\mu^{02} = \omega_\mu^{03} = 0, \quad \omega_\mu^{12} = (0, 0, A, 0)\,,\\ \omega_\mu^{13} = (0, 0, 0, A\sin\theta)\,, \quad \omega_\mu^{23} = (0, 0, 0, \cos\theta)\,,\end{gathered} \tag{3.23}$$

where A and U are functions only of the 3D radius r. We use the above expressions to compute the components of the tensors $F_{\mu\nu}^a$ and $F_{\mu\nu}^{ab}$ defined by the Eqs. (3.8) and (3.9). The non-null components of these tensors are:

$$F_{10}^0 = \frac{AA' + U}{A}, \tag{3.24}$$

and respectively:

$$\begin{aligned}&F_{10}^{01} = U' + 4\lambda^2, \qquad F_{20}^{02} = A\left(U + 4\lambda^2 r\right), \qquad F_{30}^{03} = A\sin\theta\left(U + 4\lambda^2 r\right),\\ &F_{21}^{12} = \frac{-AA' + 4\lambda^2 r}{A}, \qquad F_{31}^{13} = \frac{\left(-AA' + 4\lambda^2 r\right)\sin\theta}{A},\\ &F_{32}^{23} = \left(1 - A^2 + 4\lambda^2 r^2\right)\sin\theta,\end{aligned} \tag{3.25}$$

where A' and U' denote the derivatives with respect to variable r.

Using these expressions, we obtain the following expressions for F defined in (3.11) and F_μ^a (only non-null components) given by (3.19):

$$F = -2\frac{r^2U' + 2rU - 2rAA' + 1 - A^2}{r^2} - 48\lambda^2, \tag{3.26}$$

and respectively:

$$\begin{aligned}&F_0^0 = -\frac{A\left(rU' + 2U + 12\lambda^2 r\right)}{r}, \quad F_2^2 = -\frac{rU - rAA' + 1 - A^2 + 12\lambda^2 r}{r},\\ &F_1^1 = -\frac{rU' - 2AA' + 12\lambda^2 r}{rA}, \quad F_3^3 = -\frac{rU - rAA' + 1 - A^2 + 12\lambda^2 r}{r}\sin\theta.\end{aligned} \tag{3.27}$$

The non-null components of the energy-momentum tensor T_μ^a for the electromagnetic field created by the constant charge Q are [7]:

$$\begin{aligned}&T_0^0 = \frac{1}{Kg^2}\frac{AQ^2}{32\pi^2\varepsilon_0^2 r^4}, \qquad T_1^1 = \frac{1}{Kg^2}\frac{Q^2}{32A\pi^2\varepsilon_0^2 r^4},\\ &T_2^2 = -\frac{1}{Kg^2}\frac{Q^2}{32\pi^2\varepsilon_0^2 r^3}, \qquad T_3^3 = -\frac{1}{Kg^2}\frac{Q^2}{32\pi^2\varepsilon_0^2 r^3}\sin\theta.\end{aligned} \tag{3.28}$$

Introducing all these results in Eq. (3.18), we obtain the following field equations for $e^a_\mu(x)$:

$$\frac{2AA'}{r} - \frac{1-A^2}{r^2} - 12\lambda^2 + \frac{Q^2}{r^4} = 0, \tag{3.29}$$

$$U - AA' + rU' + 12\lambda^2 r + \frac{Q^2}{r^4} = 0, \tag{3.30}$$

$$-2AA' + rU' + 12\lambda^2 r + \frac{Q^2}{r^4} = 0. \tag{3.31}$$

Here we chosen the constant K in (3.13) so that:

$$\frac{G}{4\pi K g^2 \varepsilon_0^2} = 1. \tag{3.32}$$

Now, if we use the field equations (3.21) for ω^{ab}_μ, then we obtain the following constraint on the component U:

$$U = -AA'. \tag{3.33}$$

The solution $A(r)$ of the differential equations (3.29)–(3.31) together with the constraint (3.33) for the function $U(r)$ is:

$$A^2 = 1 + \frac{\alpha}{r} + \frac{Q^2}{r^2} + \beta r^2, \tag{3.34}$$

where α and β are arbitrary constants of integration. If we chose

$$\alpha = -2m, \quad \beta = -\frac{\Lambda}{3} \tag{3.35}$$

then we obtain the Reissner-Nordstrom-deSitter solution

$$A^2 = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2, \tag{3.36}$$

We use the units such that $c = 1$, $G = 1$, $\frac{1}{4\pi\varepsilon_0} = 1$.

4 Deformed gauge fields

The gauge fields corresponding to the de Sitter gauge symmetry for the noncommutative case are denoted by $\hat{e}^a_\mu(x, \Theta)$ and $\hat{\omega}^{ab}_\mu(x, \Theta)$, generically denoted by $\hat{\omega}^{AB}_\mu(x, \Theta)$, with the obvious meaning for the indices A, B. The main idea of the Seiberg-Witten map is to expand the noncommutative gauge fields, transforming according to the noncommutative gauge algebra, in terms of commutative gauge fields, transforming under the corresponding commutative gauge algebra, in such a way that the noncommutative and commutative gauge transformations are compatible, i.e. [2]

$$\hat{\omega}^{AB}_\mu(\omega) + \delta_{\hat{\lambda}} \hat{\omega}^{AB}_\mu(\omega) = \hat{\omega}^{AB}_\mu(\omega + \delta_\lambda \omega). \tag{4.1}$$

where $\delta_{\hat{\lambda}}$ are the infinitesimal variations under the noncommutative gauge transformations and δ_λ are the infinitesimal variations under the commutative gauge transformations.

Using the Seiberg-Witten map [2], one obtains the following noncommutative corrections up to the second order [8]:

$$\omega^{AB}_{\mu\nu\rho}(x) = \frac{1}{4}\left\{\omega_\nu, \partial_\rho\omega_\mu + F_{\rho\mu}\right\}^{AB}, \tag{4.2}$$

$$\begin{aligned} \omega^{AB}_{\mu\nu\rho\lambda\tau}(x) &= \frac{1}{32}\left(-\left\{\omega_\lambda, \partial_\tau\left\{\omega_\nu, \partial_\rho\omega_\mu + F_{\rho\mu}\right\}\right\} + 2\left\{\omega_\lambda, \left\{F_{\tau\nu}, F_{\mu\rho}\right\}\right\}\right. \\ &- \left\{\omega_\lambda, \left\{\omega_\nu, D_\rho F_{\tau\mu} + \partial_\rho F_{\tau\mu}\right\}\right\} - \left\{\left\{\omega_\nu, \partial_\rho\omega_\lambda + F_{\rho\lambda}\right\}, \left(\partial_\tau\omega_\mu + F_{\tau\mu}\right)\right\} + \end{aligned} \tag{4.3}$$

$$+ 2\left[\partial_\nu\omega_\lambda, \partial_\rho\left(\partial_\tau\omega_\mu + F_{\tau\mu}\right)\right]\big)^{AB},$$

where

$$\{\alpha,\beta\}^{AB} = \alpha^{AC}\,\beta_C^B + \beta^{AC}\,\alpha_C^B, \quad [\alpha,\beta]^{AB} = \alpha^{AC}\,\beta_C^B - \beta^{AC}\,\alpha_C^B \tag{4.4}$$

and

$$D_\mu F_{\rho\sigma}^{AB} = \partial_\mu F_{\rho\sigma}^{AB} + \left(\omega_\mu^{AC}\,F_{\rho\sigma}^{D\,B} + \omega_\mu^{BC}\,F_{\rho\sigma}^{D\,A}\right)\eta_{CD}. \tag{4.5}$$

The noncommutative tetrad fields were obtained in [8] in the second order of Θ in the limit $\lambda \to 0$ as:

$$\hat{e}_\mu^a\,(x,\Theta) = e_\mu^a\,(x) - i\;\Theta^{\nu\rho}\,e_{\mu\nu\rho}^a\,(x) + \Theta^{\nu\rho}\,\Theta^{\lambda\tau}\;e_{\mu\nu\rho\lambda\tau}^a\,(x) + O\left(\Theta^3\right), \tag{4.6}$$

where

$$\begin{aligned}
e_{\mu\nu\rho}^a &= \frac{1}{4}\left[\omega_\nu^{a\,c}\partial_\rho e_\mu^d + \left(\partial_\rho\omega_\mu^{a\,c} + F_{\rho\mu}^{a\,c}\right)\,e_\nu^d\right]\,\eta_{c\,d},\\
e_{\mu\nu\rho\lambda\tau}^a &= \frac{1}{32}2\left\{F_{\tau\nu}, F_{\mu\rho}\right\}^{a\,b}\;e_\lambda^c - \omega_\lambda^{a\,b}\left(D_\rho\,F_{\tau\mu}^{c\,d} + \partial_\rho\,F_{\tau\mu}^{c\,d}\right)\,e_\nu^m\,\eta_{d\,m} -\\
&\quad - \left\{\omega_\nu, \left(D_\rho F_{\tau\mu} + \partial_\rho F_{\tau\mu}\right)\right\}^{a\,b}\;e_\lambda^c - \partial_\tau\left\{\omega_\nu, \left(\partial_\rho\,\omega_\mu + F_{\rho\mu}\right)\right\}^{a\,b}\,e_\lambda^c -\\
&- \quad \omega_\lambda^{a\,b}\,\partial_\tau\left(\omega_\nu^{c\,d}\,\partial_\rho e_\mu^m + \left(\partial_\rho\,\omega_\mu^{c\,d} + F_{\rho\mu}^{c\,d}\right)\,e_\nu^m\right)\,\eta_{dm} + 2\,\partial_\nu\omega_\lambda^{a\,b}\partial_\rho\partial_\tau\,e_\mu^c -\\
&\quad 2\,\partial_\rho\left(\partial_\tau\,\omega_\mu^{a\,b} + F_{\tau\mu}^{a\,b}\right)\,\partial_\nu\,e_\lambda^c - \left\{\omega_\nu, \left(\partial_\rho\omega_\lambda + F_{\rho\lambda}\right)\right\}^{a\,b}\,\partial_\tau\,e_\mu^c -\\
&\quad \left(\partial_\tau\,\omega_\mu^{a\,b} + F_{\tau\mu}^{a\,b}\right)\,\left(\omega_\nu^{c\,d}\partial_\rho e_\lambda^m + \left(\partial_\rho\,\omega_\lambda^{c\,d} + F_{\rho\lambda}^{c\,d}\right)\,e_\nu^m\,\eta_{d\,m}\right)\eta_{b\,c}.
\end{aligned} \tag{4.7}$$

Using the hermitian conjugate $\hat{e}_\mu^{a\dagger}\,(x,\Theta)$ of the deformed tetrad fields given by,

$$\hat{e}_\mu^{a\,\dagger}\,(x,\Theta) = e_\mu^a\,(x) + i\;\Theta^{\nu\rho}\,e_{\mu\nu\rho}^a\,(x) + \Theta^{\nu\rho}\Theta^{\lambda\tau}e_{\mu\nu\rho\lambda\tau}^a\,(x) + O\left(\Theta^3\right), \tag{4.8}$$

a real deformed metric was introduced in [35] by the formula:

$$\hat{g}_{\mu\nu}\,(x,\Theta) = \frac{1}{2}\,\eta_{a\,b}\,\left(\hat{e}_\mu^a * \hat{e}_\nu^{b\,\dagger} + \hat{e}_\mu^b * \hat{e}_\nu^{a\,\dagger}\right). \tag{4.9}$$

We can see that this metric is, by definition, a real one, even if the deformed tetrad fields $\hat{e}_\mu^a\,(x,\Theta)$ are complex quantities.

5 Second order corrections to Schwarzschild solution

Using the ansatz (3.22) – (3.23), we can determine the deformed Schwarzschild metric. To end this, we have to obtain first the corresponding components of the tetrad fields $\hat{e}_\mu^a\,(x,\theta)$ and their complex conjugated $\hat{e}_\mu^{a+}\,(x,\theta)$ given by the Eqs. (4.6) and (4.8). With the definition (4.9) it is possible then to obtain the components of the deformed metric $\hat{g}_{\mu\nu}\,(x,\theta)$.

To simplify the calculations, we choose the parameters $\theta^{\mu\nu}$as:

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & \theta & 0 & 0\\ -\theta & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mu,\nu = 1,2,3,0. \tag{5.1}$$

Here the constant quantity θ, which determine noncommutativity of the space-time coordinates, is chosen so that it has the dimension L (length).

The non-null components of tetrad fields $\hat{e}_\mu^a\,(x,\theta)$ are:

$$\hat{e}_1^1 = \frac{1}{A} + \frac{A''}{8}\,\theta^2 + O\left(\theta^3\right), \tag{5.2}$$

$$\hat{e}_2^1 = -\frac{i}{4}\left(A + 2\,r\,A'\right)\;\theta + O\left(\theta^3\right), \tag{5.3}$$

$$\hat{e}_2^2 = r + \frac{1}{32}\left(7A\,A' + 12\,r\,A'^2 + 12\,r\,A\,A''\right)\theta^2 + O\left(\theta^3\right), \tag{5.4}$$

$$\hat{e}_3^3 = r\,\sin\,\theta\frac{i}{4}(\cos\;\theta)\;\theta + \frac{1}{8}\left(2\,r\,A'^2 + r\,A\,A'' + 2\,A\,A' - \frac{A'}{A}\right)(\sin\;\theta)\;\theta^2 \tag{5.5}$$
$$+O\left(\theta^3\right),$$

$$\hat{e}_0^0 = A + \frac{1}{8}\left(2\,r\,A'^3 + 5\,r\,A\,A'\,A'' + r\,A^2\,A''' + 2\,A\,A'^2 + A^2\,A''\right)\theta^2 \tag{5.6}$$
$$+O\left(\theta^3\right).$$

where A', A'', A''' are respectively first, second and third derivatives of $A(r)$. The complex conjugated components can be easily obtained from these expressions.

Then, using the definition (4.9), we obtain the following non-null components of the deformed metric $\hat{g}_{\mu\nu}(x,\theta)$ up to the second order:

$$\hat{g}_{1\,1}(x,\theta) = \frac{1}{A^2} + \frac{1}{4}\frac{A''}{A}\theta^2 + O\left(\theta^4\right), \tag{5.7}$$

$$\begin{aligned}
\hat{g}_{22}(x,\theta) &= r^2 + \frac{1}{16}\left(A^2 + 11\,r\,A\,A' + 16\,r^2\,A'^2 + 12\,r^2 A\,A''\right)\theta^2 + O\left(\theta^4\right),\\
\hat{g}_{33}(x,\theta) &= r^2\,\sin^2\,\theta
\end{aligned}$$
$$+\frac{1}{16}\left[4\left(2\,r\,A\,A' - r\frac{A'}{A} + r^2\,A\,A'' + 2\,r^2\,A'^2\right)\sin^2\,\theta + \cos^2\theta\right]\theta^2 + O\left(\theta^4\right)$$
$$+O\left(\theta^4\right),$$
$$\hat{g}_{00}(x,\theta) = -A^2$$
$$-\frac{1}{4}\left(2\,r\,A\,A'^3 + r\,A^3\,A''' + A^3\,A'' + 2\,A^2\,A'^2 + 5\,r\,A^2\,A'\,A''\right)\theta^2 + O\left(\theta^4\right).$$

For $\theta \to 0$ we obtain the commutative Schwarzschild solution with $A^2 = 1 + \frac{\alpha}{r}$ [see Eq. (3.34) with $Q = 0, \beta = 0$].

It is interesting to remark that, if we choose the parameters $\theta^{\mu\nu}$ as in (5.1), then the deformed metric $\hat{g}_{\mu\nu}(x,\theta)$ is diagonal as it is in the commutative case. But, in general, for arbitrary $\theta^{\mu\nu}$, the deformed metric $\hat{g}_{\mu\nu}(x,\theta)$ is not diagonal even if the commutative (non-deformed) one has this property. Therefore, we can conclude that the noncommutativity modifies the structure of the gravitational field.

For the Schwarzschild solution we have:

$$A(r) = \sqrt{1 + \frac{\alpha}{r}},\quad \alpha = -\,\frac{2\,G\,M}{c^2}; \tag{5.8}$$

The function $A(r)$ is non-dimensional, but its derivatives A', A'' and A''' have respectively the dimensions L^{-1}, L^{-2} and L^{-3}. As a consequence, all the components of the deformed metric $\hat{g}_{\mu\nu}(x,\theta)$ in (5.7) have the correct dimensions.

Now, if we introduce (5.8) in (5.7), then we obtain the deformed Schwarzschild metric. Its non-null components are:

$$\begin{aligned}
\hat{g}_{11} &= \frac{1}{1-\frac{\alpha}{r}} - \frac{\alpha\,(4r-3\alpha)}{16\,r^2\,(r-\alpha)^2}\theta^2 + O(\theta^4),\\
\hat{g}_{22} &= r^2 + \frac{2r^2 - 17\,\alpha r + 17\alpha^2}{32r\,(r-\alpha)}\theta^2 + O(\theta^4),\\
\hat{g}_{33} &= r^2\,\sin^2\,\theta + \frac{\left(r^2 + \,\alpha r - \alpha^2\right)\cos^2\,\theta - \alpha\,(2r-\alpha)}{16\,r\,(r-\alpha)}\theta^2 + O(\theta^4),
\end{aligned} \tag{5.9}$$

$$\hat{g}_{00} = -\left(1 - \frac{\alpha}{r}\right) - \frac{\alpha\,(8r - 11\alpha)}{16\,r^4}\theta^2 + O(\theta^4).$$

We can evaluate then the contributions of these corrections to the tests of General Relativity. For example, if we consider the red shift of the light propagating in a gravitational field [27, 35], then we obtain for the case of the Sun:

$$\frac{\Delta\,\lambda}{\lambda} = \frac{\alpha}{2R} - \frac{\alpha\,(8R{-}11\alpha)}{32R^4}\,\theta^2 + O\left(\theta^4\right), \tag{5.10}$$

where R is the radius of the Sun. Because for the Sun we have $\alpha = \frac{2GM}{c^2} = 2.95 \cdot 10^3\,m$ and $R = 6.955 \cdot 10^8\,m$, then we obtain from (5.10):

$$\frac{\Delta\,\lambda}{\lambda} = 2 \cdot 10^{-6} - 2.19 \cdot 10^{-24}\,\theta^2 + O\left(\theta^4\right). \tag{5.11}$$

The noncommutativity correction has a value that is with about 18 orders less than that which result from General Relativity. Therefore, presently it is impossible to verify experimentally the noncommutativity correction to the red shift test of General Relativity.

6 Corrections to the Reissner-Nordström solution

The results from previous sections apply to any spherically gravitational field having the gauge potentials defined as in Eqs. (3.22) and (3.23). In particular, they can be used also to the Reissner-Nordström-de-Sitter metric, with the function $A\,(r)$ given by (3.36). Inserting this expression of $A\,(r)$ into the equations (5.7), we obtain the deformed Reissner-Nordström-de-Sitter metric:

$$\hat{g}_{11} = \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1} + \frac{(-2mr^3 + 3m^2r^2 + 3Q^2r^2 - 6mQ^2r + 2Q^4)}{4r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)^2}\Theta^2 \tag{6.1}$$

$$+\frac{\frac{\Lambda^2r^8}{3} - \frac{3\Lambda r^6}{4} + \frac{11m\Lambda r^5}{4} - \frac{7Q^2\Lambda r^4}{3}}{4r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)^2}\Theta^2,$$

$$\hat{g}_{22} = r^2 + \frac{(r^4 - 17mr^3 + 34m^2r^2 + 27Q^2r^2 - 75mQ^2r + 30Q^4)}{16r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)}\Theta^2 \tag{6.2}$$

$$+\frac{\frac{56\Lambda^2r^8}{9} - \frac{38\Lambda r^6}{3} + 24m\Lambda r^5 - \frac{46Q^2\Lambda r^4}{3}}{16r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)},$$

$$\hat{g}_{33} = r^2\sin^2\,\theta + \frac{\cos^2\theta(r^4 + 2mr^3 - 7Q^2r^2 - 4m^2r^2 + 16mQ^2r - 8Q^4)}{16r^2(r^2 - 2mr + Q^2\frac{\Lambda}{3}r^4)}\Theta^2$$

$$+\frac{(-4mr^3 + 4m^2r^2 + 8Q^2r^2 - 16mQ^2r + 8Q^4)}{16r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)}\Theta^2 + \tag{6.3}$$

$$\frac{\sin^2\theta\left(\frac{25\Lambda^2r^6}{9} + \frac{7\Lambda r^4}{3} - \frac{14m\Lambda r3}{3} + \Lambda Q^2r^2\right)}{16r^2(r^2 - 2mr + Q^2 - \frac{\Lambda}{3}r^4)},$$

$$\hat{g}_{00} = -\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2\right) - \frac{(4mr^3 - 9Q^2r^2 - 11m^2r^2 + 30mQ^2r - 14Q^4)}{4r^6}\Theta^2 \tag{6.4}$$

$$-\frac{25\Lambda^2r^4 - 9\Lambda r^2 + 6m\Lambda r - 9\Lambda Q^2}{144r^2}\Theta^2.$$

It will be very interesting to study the gravitational singularities of the deformed scalar curvature using these considerations. Some results are given in Ref. [28].

The expression (6.4) can be used to obtain corrections to the thermodynamical quantities due to the space-time noncommutativity. If we consider the radius of event horizon for Reissner-Nordström metric

$$r_0 = m \pm \sqrt{m^2 - Q^2}, \tag{6.5}$$

then we can consider its expression as a function of Λ of the form

$$r = r_0 + a\Lambda + b\Lambda^2 + \cdots \tag{6.6}$$

because for $\Lambda = 0$ we must obtain the Reissner-Nordström solution. Inserting (6.6) into the equation

$$1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2 = 0, \tag{6.7}$$

we obtain the following expression for unknown coefficients a and b in (6.6)

$$a = \frac{r_0^5}{6\,(m r_0 - Q^2)}, \quad b = \frac{r_0^9\,(6 m r_0 - 7Q^2)}{72\,(m r_0 - Q^2)^3}. \tag{6.8}$$

In the noncommutative case, we can consider the corrected event horizon radius up to the second order as

$$\hat{r} \; = A + B\ \Theta + C\ \Theta^2. \tag{6.9}$$

substituting this expression into the equation $\hat{g}\left(\hat{r}, \Theta\right) = 0$,we obtain the corrected cosmological and black hole (Killing) event horizon radii respectively as solutions of this equation [28]:

$$\begin{aligned} \hat{r}_1 \; &= m + \sqrt{m^2 - Q^2} + a\Lambda + b\Lambda^2 \\ + &\frac{6m^4 + \sqrt{m^2 - Q^2}\left(6m^3 - 8mQ^2\right) - 11m^2Q^2 + 5Q^4}{8\left(8m^5 + \sqrt{m^2 - Q^2}\,(8m^4 - 8m^2Q^2 + Q^4) - 12m^3Q^2 + 4mQ^4\right)}\Theta^2, \end{aligned} \tag{6.10}$$

and

$$\begin{aligned} \hat{r}_2 \; &= m - \sqrt{m^2 - Q^2} + a\Lambda + b\Lambda^2 + \\ + &\frac{6m^4 - \sqrt{m^2 - Q^2}\left(6m^3 - 8mQ^2\right) - 11m^2Q^2 + 5Q^4}{8\left(8m^5 - \sqrt{m^2 - Q^2}\,(8m^4 - 8m^2Q^2 + Q^4) - 12m^3Q^2 + 4mQ^4\right)}\Theta^2. \end{aligned} \tag{6.11}$$

The distance between the corrected event horizon radii is given by following relation in an example case, when $m = 2Q$

$$\hat{d} = \hat{r}_1 - \hat{r}_2 = 2\sqrt{3}Q + \frac{51\sqrt{3}}{4Q}\Theta^2 \equiv d + \Delta d. \tag{6.12}$$

Therefore, in the noncommutative space-time the distance between horizons is more than in commutative case. Then, we obtain from (6.12)

$$\frac{\Delta d}{d} = \frac{51\Theta^2}{8Q^2}. \tag{6.13}$$

The ratio of this change due to the noncommutativity correction to horizon radii has a value which is much too small to be experimentally detected.

Having the horizon radii determined, it is possible then to obtain corrections for the temperature, horizon area and entropy of the black hole [24, 25, 26, 28].

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