GENERAL RELATIVISTIC ANALOG SOLUTIONS FOR YANG-MILLS THEORY ON NONCOMMUTATIVE SPACE-TIME

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A model of gauge theory with U(2) as local symmetry group is developed over a noncommutative space-time. The integral of the action is written considering a gauge field coupled with a Higgs multiplet. The gauge fields are calculated up to the first order in the noncommutativity parameter using the equations of motion and Seiberg-Witten map. The solutions are determined supposing that in zero-th order they have a general relativistic analog form. The Wu-Yang ansatz for the gauge fields is used to solve the field equations. Some comments on the quantization of the electrical and magnetical charges are also given, with a comparison between commutative and noncommutative cases.

1. INTRODUCTION

Noncommutative (NC) gauge theories have been intensively studied during the past years, prompted by the results of Seiberg and Witten's seminal paper [1], connecting commutative and NC fields. Recent results [2–4] show that NC space-time might be endowed with a deformed symmetry structure. It has been established in a series of papers [5–7] that the Seiberg-Witten (SW) results can be obtained in an entirely independent way of string theory. The corresponding method uses only algebraic properties of the canonical NC space-time via properties of the $\star$ – product between functions defined on this space.

In Ref. [8], a BPS monopole (Bogomolnyi, Prasad, Sommerfield [17, 18]) solution was considered at the first order in the NC parameter $\theta^{\mu\nu}$, and the extension of the results up to second order is given in Ref. [9]. The analysis using the NC eigenvalue equation for scalar field successfully reproduced the D-string picture. Study of NC monopoles using SW method was carried out in Refs. [10–13]. There, the solutions were transformed in the commutative description via SW map. More recently [14], an explicit classical dyon solution for NC version of the Yang-Mills-Higgs (YMH) model was studied using BPS equation.

Finding solutions for Yang-Mills (YM) theories is usually difficult because the field equations are non-linear. However, gauge theories and general relativity share some mathematical similarities, and this connection can be used to find solutions to one theory knowing solutions of the other theory and reciprocal [15]. In our paper we extend these results to the NC case, considering U(2) as gauge group. In section 2, we obtain the general relativistic analog solutions for the commutative U(2) gauge theory coupled with a Higgs multiplet in adjoint representation. The field equations are written and their solutions are obtained. Besides BPS monopole, we consider the Schwarzschild and de-Sitter dyon solutions. The "electromagnetic" features of these solutions are investigated by using 't Hooft's definition of the generalized, gauge invariant, U(1) field strength tensor [16].

Section 3 is devoted to the NC U(2) gauge theory. Our analysis is based on field equations and SW map. For the monopole solution one considers also the BPS equation and the NC field components are obtained by expanding them in powers of θ. The NC general relativistic analog solutions are obtained both for monopole and dyon solution and the Witten effect [14, 25] is investigated. We show that the relation between classical electric and magnetic charges also holds in NC space-time. Some possible extensions of these results are also suggested.

2. GAUGE FIELDS ON COMMUTATIVE SPACE-TIME

We will consider first the commutative U(2) gauge theory coupled to a Higgs multiplet $\Phi(x)$ in adjoint representation. The action for this Yang-Mills Higgs (YMH) system is [9, 14]

$$S = tr \int d^4x \left(-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi D^\mu \Phi \right). \tag{2.1}$$

Gauge fields $A_\mu(x) = A_\mu^A(x) T_A$, $\mu = 1, 2, 3, 0$ take values in the Lie algebra of $U(2)$, with generators T_A, $A = 0, 1, 2, 3$ defined as

$$T_0 = \frac{1}{2} I, \quad T_a = \frac{1}{2} \sigma_a, \quad a = 1, 2, 3, \tag{2.2}$$

where I is the 2×2 unity matrix and σ_a are the Pauli matrices. Analogous, $\Phi(x) = \Phi^A(x) T_A$ is the Higgs multiplet whose components transform under the adjoint representation of the $U(2)$ group. Therefore, the components $\left(\Phi^0, A_\mu^0\right)$ correspond to $U(1)$ sector, and $\left(\Phi^a, A_\mu^a\right)$ – to $SU(2)$ sector. Note that the group $SU(2)$ is not allowed here since the algebra of any special unitary group is not closed when multiplication is defined by the $*$-product. In such cases we have to work with the associative Hopf algebras. The gauge theory is defined over the Minkowski space-time endowed with the metric

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2\,. \tag{2.3}$$

whose components are $\eta_{\mu\nu} = diag(1,1,1,-1)$.

Gauge covariant derivative and field strength tensor of the gauge fields are defined as follows

$$D_\mu \Phi = \partial_\mu \Phi + g\left[A_\mu, \Phi\right], \tag{2.4}$$

and respectively

$$F_{\mu\nu} = \partial_\mu A_\nu = \partial_\nu A_\mu + g\left[A_\mu, A_\nu\right], \tag{2.5}$$

where g is the gauge coupling constant.

The general equations of motion for YMH system are

$$\partial^\nu F^A_{\mu\nu} = f^A_{BC}\left[F^A_{\mu\nu} A^{C\nu} - \left(D_\mu \Phi^B\right)\Phi^C\right], \tag{2.6}$$

$$\partial^\mu D_\mu \Phi^A = g f^A_{BC}\left(D_\mu \Phi^B\right) A^{C\mu}\,, \tag{2.7}$$

where $f^A_{BC} = -f^A_{CB}$ are the $U(2)$ constants of structure and $F^A_{\mu\nu}$ are the components of the strength tensor $F_{\mu\nu} = F^A_{\mu\nu} T_A$ with values in the Lie algebra. In order to obtain the solutions of the equations (2.6)–(2.7) we adopt the case with vanishing fields of $U(1)$ sector, *i.e.* $\Phi^0 = 0$, $A^0_\mu = 0$. Also, we consider the generalized Wu-Yang ansatz [15]

$$\begin{aligned} A^a_i &= \varepsilon_{aij}\frac{x_j}{gr^2}\left[1 - K(r)\right] + \left(\frac{x_a x_i}{r^2} - \delta_{ia}\right)\frac{G(r)}{gr}, \quad i,j = 1,2,3, \\ A^a_0 &= \frac{x_a}{gr^2} J(r), \\ \Phi^a &= \frac{x_a}{gr^2} H(r), \end{aligned} \tag{2.8}$$

where $K(r)$, $G(r)$, $J(r)$, and $H(r)$ are the ansatz functions to be determined by the equations of motion. Also, the constants of structure f^a_{bc} of the $SU(2)$ group have been identified with the components of the complete antisymmetric tensor ε_{abc}, with $\varepsilon_{123} = +1$. Introducing the expressions (2.8) in (2.6)–(2.7), we obtain the following set of coupled, non-linear YMH field equations

$$r^2 K'' = K\left(K^2 + G^2 + H^2 - J^2 - 1\right),$$

$$r^2G'' = G\left(K^2 + G^2 + H^2 - J^2 - 1\right),$$
$$r^2J'' = 2J\left(K^2 + G^2\right), \tag{2.9}$$
$$r^2H'' = 2H\left(K^2 + G^2\right),$$

where K'', G'', J'' and H'' denote differentiation with respect to r.

One of the most important solution to the equations (2.9) is

$$K(r) = \cos(\alpha)\, r\, \mathrm{csc}h(r), \quad G(r) = \sin(\alpha)\, r\, \mathrm{csc}h(r),$$
$$J(r) = \sinh(\beta)\left[1 - r\coth(r)\right], \quad H(r) = \cosh(\beta)\left[1 - r\coth(r)\right], \tag{2.10}$$

where α and β are arbitrary constants. It was discovered by Bogomolnyi [17] and independently by Prasad and Sommerfield [18], being named BPS solution. This solution satisfies, besides the YM field equations (2.9), the BPS-equation

$$D_i\Phi + \frac{1}{2}\varepsilon_{ijk}\, F_{jk} = 0. \tag{2.11}$$

It is important remark that not any solution of the YM field equations (2.9) verifies also the BPS equation (2.11).

In order to simplify the above results, we will consider now the case of monopole solutions, *i.e.* we suppose $J(r) = G(r) = 0$, and introduce the notations

$$W(r) = \frac{1 - K(r)}{gr}, \quad F(r) = \frac{H(r)}{gr}. \tag{2.12}$$

Then, the ansatz (2.8) becomes

$$A_i^a = \varepsilon_{aij}\frac{x_j}{gr}W(r), \quad \Phi^a = \frac{x_a}{gr}F(r), \tag{2.13}$$

and the BPS-equation (2.11) implies

$$r\left(F' - 2W'\right) + 2(F - 2W) + r\,W\left(W - 2F\right) = 0. \tag{2.14}$$

It is easy to show that the solution (2.10) verifies this equation, while our general relativistic solutions [see (2.16) and (2.18)] do not. This means that the last ones are monopole but not BPS solutions.

The non-linear nature of the YM field equations makes finding solutions difficult. However, because the general relativity is also a non-Abelian gauge theory [19–21], there is a mathematical connection between the two theories. As a consequence, it is possible to find solutions to the field equations of the gauge theory starting from those of the general relativity. We will give now some general relativistic analog solutions for YMH system.

We begin by examining the Schwarzschild solution of general relativity which has two non-trivial components to the metric tensor

$$g_{11}=-\frac{1}{g_{00}}=\left(1-\frac{2GM}{c^2r}\right)^{-1}. \tag{2.15}$$

Trying this form of g_{11} in the equations (2.9) one immediately finds the following solution

$$K(r)=\frac{\cos(\alpha)}{1-\frac{2GM}{c^2r}},\quad G(r)=\frac{\sin(\alpha)}{1-\frac{2GM}{c^2r}},$$

$$J(r)=-\frac{2GM}{r}\frac{\sinh(\beta)}{1-\frac{2GM}{c^2r}},\quad H(r)=-\frac{2GM}{r}\frac{\cosh(\beta)}{1-\frac{2GM}{c^2r}}, \tag{2.16}$$

where α and β are arbitrary constants.

A second example of general relativistic analog solutions for YMH system is that including a non-zero cosmological constant Λ. The time-time component of the metric tensor is [22]

$$g_{00}=1-\frac{2GM}{c^2r}-\frac{\Lambda}{3}r^2,$$

where Λ is the cosmological constant. The Newtonian potential corresponding to this solution has the expression [15]

$$\varphi(r)=\frac{g_{00}-1}{2}=-\frac{2GM}{c^2r}-\frac{\Lambda}{3}r^2. \tag{2.17}$$

Using this potential as a starting point one finds the following analog de-Sitter solution

$$K(r)=\cos(\alpha),\quad G(r)=\sin(\alpha),\quad J(r)=H(r)=\frac{A}{r}+Br^2, \tag{2.18}$$

where α, A and B are arbitrary constants.

All these solutions (2.10), (2.16) and (2.18) to the YM field equations have interesting "electromagnetic" features. To investigate these properties we will use 't Hooft's definition of generalized, gauge invariant, $U(1)$ field strength tensor [16]

$$F_{\mu\nu}=\partial_\mu\left(\varphi^a A_\nu^a\right)-\partial_\nu\left(\varphi^a A_\mu^a\right)-\frac{1}{g}\varepsilon_{abc}\varphi^a\left(\partial_\mu\varphi^b\right)\left(\partial_\nu\varphi^c\right), \tag{2.19}$$

where $\varphi^a=\Phi^a\left(\Phi^b\Phi^b\right)^{-1/2}$. This field strength tensor reduces to the usual expression if one performs a gauge transformation to the Abelian gauge

$\varphi^a = \delta^{3a}\varphi(r)$ [23]. If one associate this $U(1)$ with the photon of electromagnetism, then the above mentioned solutions carry electric and/or magnetic charges. In general, the electric and magnetic fields associated with these solutions are [16]

$$E_i = F_{0i} = \frac{x_i}{gr}\frac{d}{dr}\left(\frac{J(r)}{r}\right),$$

$$B_i = \frac{1}{2}\varepsilon_{ijk}F_{jk} = -\frac{x_i}{gr^3}. \tag{2.20}$$

If we chose $J(r) = 0$, *i.e.* $A_0^a = 0$ (see (2.8)), then the electric field vanishes and the corresponding solutions (2.10), (2.16) and (2.18) describe a point-like monopole (BPS, Schwarzschild or de-Sitter) of strength $q_m = -\frac{4\pi}{g}$. In the case when $J(r) \neq 0$, these solution describe a dyon [14] carrying both electric and magnetic charges. We remark that gauge theories coupled to Higgs fields exhibit a remarkable phenomenon, usually called Witten effect [25], related to the ϑ-angle. Indeed, suppose we add under the integral (2.1) a ϑ-term

$$\Delta L = \vartheta\frac{e^2}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}tr\left(F_{\mu\nu}F_{\rho\sigma}\right). \tag{2.21}$$

Then, denoting the electric and magnetic charges by q_e and respectively q_m, it can be shown that [14, 24]

$$q_e = ne + \frac{\vartheta e^2}{8\pi^2}q_m. \tag{2.22}$$

Identifying the coupling constant g with the electron charge – e (as in $U(1)$ gauge theory), it results

$$q_e = \left(n + \frac{\vartheta}{2\pi}\right)e. \tag{2.23}$$

This result is known as Witten effect [25] which shows that the electric charge q_e of a dyon is modified by the ϑ-term. We will show that, for NC dyons of Schwarzschild and de-Sitter type, the same formula holds as in ordinary space.

3. NONCOMMUTATIVE GAUGE THEORY

The NC structure of the space-time is determined by the (canonical) commutation relation

$$\left[x^\mu, x^\nu\right] = i\,\theta^{\mu\nu}, \tag{3.1}$$

where $\theta^{\mu\nu} = -\theta^{\nu\mu}$ is an antisymmetric constant matrix. It is well known [22] that NC field theory is constructed by introducing the star product "$\star$" between the functions $f(x)$ and $g(x)$ defined over the space-time

$$(f \star g)(x) = f(x)\, e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu} g(x). \tag{3.2}$$

Then, the action of the $U(2)$-YMH system considered in Section 2 extends to the NC form

$$S_{NC} = tr \int d^4x \left(-\frac{1}{2} \hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu} + D_\mu \hat{\Phi} \star D^\mu \hat{\Phi} \right). \tag{3.3}$$

The covariant derivative and field strength tensor are defined in the following NC forms

$$D_\mu \hat{\Phi} = \partial_\mu \hat{\Phi} + g\left[\hat{A}_\mu, \hat{\Phi} \right]_\star, \tag{3.4}$$

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g\left[\hat{A}_\mu, \hat{A}_\nu \right]_\star, \tag{3.5}$$

where we have denoted

$$\left[\hat{A}(x), \hat{B}(x) \right] = \hat{A}(x) \star \hat{B}(x) - \hat{B}(x) \star \hat{A}(x).$$

The NC general relativistic analog solutions can be obtained by expanding the fields $\hat{A}_i$ and $\hat{\Phi}$ in powers of θ

$$\hat{A}_i = \left(\hat{A}_i^{a(0)} + \hat{A}_i^{a(1)} + \hat{A}_i^{a(2)} + \ldots \right) T_a + \left(\hat{A}_i^{0(1)} + \hat{A}_i^{0(2)} + \ldots \right) T_0, \tag{3.6}$$

$$\hat{\Phi} = \left(\hat{\Phi}^{a(0)} + \hat{\Phi}^{a(1)} + \hat{\Phi}^{a(2)} + \ldots \right) T_a + \left(\hat{\Phi}^{0(1)} + \hat{\Phi}^{0(2)} + \ldots \right) T_0, \tag{3.7}$$

where the superscript (n) denotes the order of θ. As the solutions at $O(\theta^0)$ order, we adopt the monopole solutions with vanishing $U(1)$ components $\hat{A}_i^{0(0)} = \hat{\Phi}^{0(0)} = 0$ and the $SU(2)$ components given in (2.13). The NC dyon solutions can be obtained on an analogous way. In addition, we take only space-space noncommutativity, $\theta^{0i} = 0$ (due to the known problem with unitarity) and choose the coordinate system so that the parameters $\theta^{\mu\nu}$ are given as

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mu, \nu = 1, 2, 3, 0. \tag{3.8}$$

The BPS NC monopole solution has been obtained by solving order by order the BPS equation [12–14]

$$D_i\hat{\Phi} + \frac{1}{2}\varepsilon_{ijk}\,\hat{F}^{jk} = 0. \tag{3.9}$$

The only non-zero component at the first order $O(\theta)$ is

$$\hat{A}_i^{0(1)} = \theta_{ij}\,x^j\,\frac{1}{4r^2}W(W+2F), \qquad \hat{\Phi}^{0(1)} = 0. \tag{3.10}$$

The BPS monopole solution at the $O(\theta^2)$ order are given in Appendix. Because our general relativistic analog solutions do not satisfy the BPS equation, we will use the field equations and SW map [1] as an alternative way.

The main idea of the SW map is to expand the NC fields in terms of commutative fields in such a way that the gauge transformations $\delta_{\hat{\lambda}}$ and δ_λ are compatible, *i.e.*

$$\hat{A}_\mu(A) + \delta_{\hat{\lambda}}\hat{A}_\mu(A) = \hat{A}_\mu(A + \delta_\lambda A),$$

where

$$\delta_\lambda A = [\lambda, A], \quad \delta_{\hat{\lambda}}\hat{A} = \left[\hat{\lambda}, \hat{A}\right]_*, \quad \lambda = \lambda^A T_A, \quad \hat{\lambda} = \hat{\lambda}^A T_A.$$

This map is derived from the requirement of gauge equivalence of the two descriptions. Using this condition, one obtain up to the first order in θ [1, 26, 27]

$$\hat{A}_\mu = A_\mu - \frac{1}{4}\theta^{\nu\rho}\{A_\nu, \partial_\rho A_\mu + F_{\rho\mu}\} + O(\theta^2), \tag{3.11}$$

$$\hat{\Phi} = \Phi - \frac{1}{4}\theta^{\mu\nu}\{A_\mu, \partial_\nu (D_\nu + \partial_\nu)\Phi\} + O(\theta^2), \tag{3.12}$$

where the Higgs multiplet Φ transforms under the adjoint representation of $U(2)$ group.

Choosing the fields at the $O(\theta^0)$ order as in (2.13), we obtain from (3.11)–(3.12) the following non-zero components at the first order in θ

$$\hat{A}_i^{0(1)} = \theta_{ij}\,x^j\,\frac{1}{4r^2}W(W+2rW'), \tag{3.13}$$

$$\hat{\Phi}^{0(1)} = -\theta_{ij}\varepsilon^{ijk}x_k\,\frac{1}{4r^2}WF(2-rW) \tag{3.14}$$

Note that the NC fields in $U(1)$ sector have non-zero components at the $O(\theta)$ order, although their components in the zeroth order vanish. We remark also that the results obtained by SW map method differ from those deduced by using BPS equation. This is due to the fact that the NC gauge fields obtained by SW map are solutions of the field equations but not for BPS equation.

In order to analyze the Noether charge of our solutions, we consider the NC version of the action (3.3) including an additional ϑ-term

$$S_{NC} = tr\int d^4x\left(-\frac{1}{2}F_{\mu\nu}\star F^{\mu\nu} + D_\mu\Phi\star D^\mu\Phi + \vartheta\frac{e^2}{16\pi^2}F_{\mu\nu}\star\tilde{F}^{\mu\nu}\right). \quad (3.15)$$

The use of the equations of motion allows to explicitly find the component J^0 of the current J^μ of the gauge fields [14]

$$J^0 = -\frac{1}{e}tr\left(2F^{0i}\star D_i\Phi - \frac{\vartheta e^2}{4\pi^2}\tilde{F}^{0i}\star D_i\Phi\right). \quad (3.16)$$

Then, the conserved charge of the general relativistic analog dyon is

$$N = \int d^3xJ^0 = -\frac{1}{e}tr\int d^3x\left(2F^{0i}\star D_i\Phi - \frac{\vartheta e^2}{8\pi^2}\varepsilon^{ijk}F_{jk}\star D_i\Phi\right), \quad (3.17)$$

or after integration

$$N = -\frac{1}{e}Q + \frac{\vartheta e}{8\pi^2}M. \quad (3.18)$$

Here, Q and M are the NC electric and respectively magnetic charges of the dyon, defined as [14]

$$Q = 2\,tr\int d^3xE_i\star D_i\Phi, \quad (3.19)$$

$$M = 2\,tr\int d^3xB_i\star D_i\Phi. \quad (3.19)$$

Supposing that the charge N is quantized in integer units n, we obtain from (3.18)

$$q_e = ne + \frac{\vartheta e^2}{8\pi^2}q_m. \quad (3.26)$$

This is, we obtained for NC general relativistic analog dyon the same formula that holds for the case of ordinary space, *i.e.* the equation (2.22).

In summary, we have constructed explicit noncommutative monopole and dyon solution starting from the general relativistic analog results. We showed that noncommutative fields in $U(1)$ sector have non-zero components at the first order in parameter θ although in zeroth order they are vanishing. Moreover, after extending the Noether approach to the case of noncommutative gauge theory, we have proven that the noncommutativity of space-time [relation (3.1)] do not change the Witten effect. We found that the dyon's charge shift is independent on the parameters $\theta^{\mu\nu}$ of noncommutativity. It should be interesting to investigate the general relativistic analog monopole and dyon solutions for the case when $\theta^{\mu\nu} = \theta^{\mu\nu}(x)$ because this coordinate dependence may introduce definite changes in the charge shift due to noncommutativity of space-time.

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