CLASSICAL AND QUANTUM MODELS FOR GAUGE FIELDS
- Stage 4 -

Objective 1. Developing analytical computing programs for studying the gauge fields defined over commutative and non-commutative space-times.

1.1. Elaborating of computing routines for the gauge field tensor

A. The case of gauge fields defined over commutative space-times

We consider a system of matter fields $\phi_i(x)$, $i = 1,2,...,m$, $x = (x^\mu)$-the coordinates on the space-time *M*, described by the Lagrangian $L = L(\phi_i, \partial_\mu \phi_i)$, $\mu = 0,1,2,...,D-1$, which is global invariant under the transformations of a Lie group of symmetry *G*. The infinitesimal generators T_a, $a = 1,2,...,n$ satisfy the equations

$$[T_a, T_b] = i\, f_{ab}^c T_c, \tag{1}$$

where $f_{ab}^c = -f_{ba}^c$ are the constants of structure of the group *G*, and the fields $\phi_i(x)$ belong to a *m*-dimensional representation of *G*. If we suppose that the group *G* is also a gauge (local) group of symmetry, then we must introduce the associated gauge fields $A_\mu^a(x)$, which are the components of a Lie algebra-valued 1-form $A = A_\mu^a(x) T_a dx^\mu$. We define then the curvature 2-form

$$F = dA - \frac{i}{2}[A, A], \tag{2}$$

and we obtain the following expressions of its components

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\, f_{bc}^a A_\mu^b A_\nu^c, \tag{3}$$

where *g* is the gauge coupling constant.

The integral action of the gauge fields $A_\mu^a(x)$ is

$$S_g = -\frac{1}{4}\int d^4x\, F_{\mu\nu}^a F_a^{\mu\nu}, \tag{4}$$

and it allows us to obtain the field equations by imposing the minimum action principle.

For any effectively application we need to calculate first of all the components $F_{\mu\nu}^a(x)$ of the tensor associated to the gauge fields $A_\mu^a(x)$. Obtaining these components is laborious and this imposes us the use of analytical computing programs. For such a purpose we developed some computing routines which are adequate for running under MAPLE Program. They allow to introduce the components of the gauge fields corresponding to different values of the space index μ and the group index *a*, this operation depending therefore on the dimensions of space-time *M* and the gauge group *G*.

If the dimension of the considered gauge group is equal to that of the space-time, then the components of the gauge fields (potentials) $A_\mu^a(x)$ can be introduced directly with the instructions:

```
> grdef(`A{^a miu niu)`); > grcalc(A(up,dn,dn));> grdisplay(_);
```

Having these components, the program allows us to compute then the components $F_{\mu\nu}^a$ associated to the gauge fields, by using the definition (3):

```
> grdef(` f {^a b c}`);
```

>grdef(`*F* {^*a miu niu*}:= *A* {^*a niu*, *miu*}- *A* {^*a miu*, *niu*}- *f* {^*a b c*} * *A* {^*b miu*} *A* {^*c miu*}`);

If the dimension of the gauge group differs of that of the space-time, the different components are introduced individually for any gauge field, i.e. $A_\mu^1, A_\mu^2, ..., A_\mu^n$ and respectively $F_{\mu\nu}^1, F_{\mu\nu}^2, ..., F_{\mu\nu}^n$, by the same instructions. However, in our works we developed a computing program which allows us to introduce directly these quantities, even in the case when the two dimensions are different [4]. For such a purpose we used the possibility of working with strings in the MAPLE program. For example, the components of the gauge $A_\mu^a(x)$ can be introduced using the following instructions:

> with(StringTools);
> for *a* from 1 to *n* do
> X = cat(`A`,*a*,`{miu}`): grdef(X): grcalc(X): grdisplay(_):
> end do;

The components of the associated tensor $F_{\mu\nu}^a(x)$ are computed then by using the definition (3) and the same facilities offerd by "String Tools". The program is presented in detail in our work [4].

As an example of application of the above procedure, we consider the model with spherical symmetry and having $G \times SU(2)$ as gauge group, i.e. the symmetry group is the direct product of the gravitational gauge G and the group $SU(2)$ [**Objective 2**].

B. The case of gauge fields defined over non-commutative space-times

In this case the coordinates of the space-time do not commute and they satisfy the relations

$$\left[x^\mu, x^\nu\right] = i\theta^{\mu\nu}(x), \tag{5}$$

where $\theta^{\mu\nu}(x) = -\theta^{\nu\mu}(x)$ is a bi-vector [2, 3]. In order to develop a gauge theory on such a space, we introduce a new product between fields or different functions, which is denoted by the symbol „$*$" and is named the *star product*. Because the components of gauge fields and the associated tensor are Lie algebra-valued differential forms, we defined the star product adequately for such quantities [2, 9]:

$$\alpha * \beta = \alpha \wedge \beta + \sum_{n=1}^{\infty}\left(\frac{i\hbar}{2}\right)^n C_n(\alpha, \beta). \tag{6}$$

Here, the symbol „$\wedge$" denotes the exterior product, and $C_n(\alpha,\beta)$ are bi-linear operators which are chosen so that the associativity property of the star product is satisfied. The gauge field tensor has an expression on the same form (2), but the commutator of two differential forms α and β is written by using the star product [2, 3]

$$[\alpha, \beta]_* = \alpha * \beta - (-1)^{|\alpha||\beta|}\beta * \alpha, \tag{7}$$

where $|\alpha|$ and $|\beta|$ denotes the rank of the two differential forms α and respectively β. In the computing routine we have to introduce first the bilinear operators $C_n(\alpha,\beta)$, then the components of the gauge fields $\hat{A}_\mu^a(x)$, and finally we computed the associated tensor $\hat{F}_{\mu\nu}^a(x)$. The hat symbol „^" over different quantities denotes their corresponding expressions in the gauge theory on the non-commutative space-time. We used the Seiberg-Witten map [2] in order to calculate these quantities, supposing that we have a $U_*(2)$ gauge theory. The corresponding expressions up to the first order $O(\hbar)$ are [2, 10]

$$A_{\mu}^{(1)} = -\frac{1}{4}\theta^{\rho\sigma}\left\{A_{\rho}, \nabla_{\sigma}A_{\mu} + F_{\sigma\mu}\right\}, \tag{8}$$

$$F_{\mu\nu}^{(1)} = -\frac{1}{4}\theta^{\rho\sigma}\left(\left\{A_{\rho}, \nabla_{\sigma}F_{\mu\nu} + D_{\sigma}F_{\mu\nu}\right\} - 2\left\{F_{\mu\rho}, F_{\nu\sigma}\right\}\right). \tag{9}$$

In our computing program, which is presented in detail in the work [7], we used the following instructions for obtaining the first order quantum corrections:

> grdef(` *A*1 {^*a miu*}:= - *theta*{^*rho* ^*sigma*} ∗ *A* {^*b rho*} ∗ ( *DA* {^*c sigma miu*}+ *F* {^*c sigma miu*}) ∗ *d* {^*a b c*}/4`);

> grdef(` *F*1 {^*a miu niu*}:= -*theta*{^*rho* ^*sigma*} ∗ (*A* {^*b rho*} ∗ (*DF* {^*c miu niu, sigma*}+ *Dg* {^*c miu niu sigma*})-2 ∗ *F* {^*b miu rho*} ∗ *F* {^*c niu sigma*}) ∗ *d* {^*a b c*}/4;

> grcalc ($A1(up, dn), F1(up, dn, dn)$);

>grdisplay(_);

Some specific applications are presented below to the **Objective 2.**

1.2. Construction of the program for obtaining the field equations

First of all, we considered the case of an internal gauge group of symmetry, for example *U*(*N*) or *SU*(*N*). Imposing the extremum condition $\delta S_g = 0$ on the integral of action (4), we obtain the following field equations

$$\partial^{\mu}F_{\mu\nu}^{a} + g f_{bc}^{a} A^{\mu b}F_{\mu\nu}^{c} = 0, \tag{10}$$

Our computing program includes instructions for introducing the gauge field components $A_{\mu}^{a}(x)$, the associated tensor $F_{\mu\nu}^{a}$, and then the field equations can be written as follows:

> grdef(`*EQ*{^*a miu*}:=*F*{^*a mu niu,^miu*}+*g*\**f*{^*a b c*}\**A*{^*b*^*miu*}\**F*{^*c miu niu*}`);

> grcalc(*EQ*(*up*,*dn*);

> grdisplay(_);

In fact, these instructions allow to compute the left-hand side of the equation (10), and the field equations are obtained by equaling the obtained expressions with zero. Secondly, we constructed a computing program where the gauge group is $G \times SU(N)$ and the results were then particularized for $N = 2$. The $SU(N)$ gauge fields are denoted by $A_{\mu}^{a}(x)$, and those corresponding to the group G by $G_{\mu}^{\alpha} = \delta_{\mu}^{\alpha} - gC_{\mu}^{\alpha}, \alpha = 0,1,\ldots,D-1$. The *SU*(*N*) gauge tensor is computed by using the expression:

$$F_{\mu\nu}^{a} = D_{\mu}A_{\nu}^{a} - D_{\nu}A_{\mu}^{a} + g_1 f_{bc}^{a}A_{\mu}^{b}A_{\nu}^{c}, \tag{11}$$

where g_1 denotes the $SU(N)$ gauge coupling constant and D_{μ} is the gauge covariant derivative. The field equations are obtained as previous by imposing the principle of minimum action for the action of the two gauge fields and using the facilities offered by "StringTools" [4]. For such a purpose, we introduced first the components of the gravitational energy-momentum tensor T_{ν}^{α} and the currents J_{a}^{μ} associated to the group *SU*(*N*) [7]. The instructions for computing the field equations are:

```
EqSUN:= proc()
  for i from 1 to m do
  X:=cat(`EQ`,i,`{nu}:=Ac`,i,`{mu nu,^mu} +
  g1*eta1{nu sigma}*J`,i,`{^sigma}`);
  grdef(X); grcalc(X); grdisplay(_);
   end do; end proc:
EqGrav:=proc()
  grdef(`gb{mu nu}:=eta1{alpha beta}*Gbinv{mu ^alpha}*
```

```
Gbinv{nu ^beta}`,sym={[1,2]}); grcalc(gb(dn,dn));
#grdef(`T{^nu alpha}:= here is the expression of
#the gravitational energy-momentum tensor
grcalc(T(up,dn));
grdef(`EX{^mu ^nu alpha}:=(1/4)*eta1inv{^mu ^rho}*
eta1inv{^nu ^sigma}*gb{alpha beta}*F{^beta rho sigma}-
(1/4)*eta1inv{^nu ^rho}*F{^mu rho alpha}+
(1/4)*eta1inv{^mu ^rho}*F{^nu rho alpha}-
(1/2)*eta1inv{^mu ^rho}*kdelta{^nu alpha}*
F{^beta rho beta}+(1/2)*eta1inv{^nu ^rho}*
kdelta{^mu alpha}*F{^beta rho beta}`);
grcalc(EX(up,up,dn));
grdef(`EQG{^nu alpha}:=EX{^mu ^nu alpha,mu}+
g*T{^nu alpha}`); grcalc(EQG(up,dn)); grdisplay(_);
end proc:
```

1.3. Elaboration of programs for obtaining solutions of the field equations

In order to obtain solutions of the field equations, we elaborated an analytical program corresponding to case of gauge group $G \times SU(2)$. The gauge fields are determined in this case only by two functions: $U(r)$ - for G and $V(r)$ - for $SU(2)$. Using our program, we obtained the following two coupled field equations:

$$(1-gU)(rV''+2V')-grVU''-2gVU'-2grU'V'=0, \qquad (12)$$

$$2grU'(1-gU)+2U-gU^2=0. \qquad (13)$$

The solutions of these equations are obtained by means of computing package *dsolve,* included in MAPLE program, as follows:

```
> grdef(`G{^alpha mu}`);
> grdef (`A{^a mu}`);
> grcalc(G(up,dn)); grcalc(A(up,dn));
> ode1:=diff(U(r), r) = (g*U(r)^2-2*U(r))/(2*g*r*(1-g*U(r)));
> dsolve(ode1, U(r));
> ode2:= (1- g*U(r))*r*diff(V(r), r, r)+2*diff(V(r), r)=g*r*V(r)*diff(U(r), r, r)+
  2*g*V(r)*diff(U(r), r)+2*g*r*diff(U(r), r)*diff(V(r), r);
> dsolve(ode2,V(r));
```

The solutions are:

$$U_{1,2}(r)=1\pm\sqrt{1+\frac{a}{r}}\,/\,g, \quad V_1(r)=C_1\sqrt{1+\frac{a}{r}}, \quad V_2(r)=C_2\sqrt{\frac{r}{r+a}}, \qquad (14)$$

where a and $C_{1,2}$ are arbitrary constants of integration. Choosing $a=-2GM$, the solution $U(r)$ will correspond the Schwarzschild metric [7].

Objective 2. Applications of the computing programs to the gauge theory with symmetry group $SU(n)\times SO(p,q)$

2.1. Applications of computing programs and routines to the gauge models with spherical symmetry

A. Commutative gauge theory. As a first applications of our computing programs, we considered the case when the gauge group is $SU(2)\times SO(4,1)$, where $SU(2)$ is an internal gauge group of symmetry and $SO(4,1)$ is the de-Sitter group. This model of gauge theory describes simultaneously the interactions between internal and gravitational gauge fields. The associated spherically symmetric gauge fields are chosen under the form:

$$e_\mu^0 = \left(\sqrt{N},0,0,0\right) e_\mu^1 = \left(\frac{1}{\sqrt{N}},0,0,0\right), e_\mu^2 = \left(0,0,r,0\right), e_\mu^3 = \left(A,0,0,r\sin\theta\right) \text{-for } SO(4,1)$$

$$A = uT_3 dt + \cos\theta T_3 d\varphi \text{ , - for } SU(2)$$

where N and u are functions only of radial variable *r*. The corresponding field equations are

$$\left(rN' + N - 1\right) + r^2 u'^2 - \frac{1}{r^2} + \Lambda r^2 = 0\,, \tag{15}$$

$$ru'' + 2u' = 0\,. \tag{16}$$

The analytical solutions were obtained by using our computing program. The instructions and results are:

```
> ode1:=r*diff(u(r),r,r)+2*diff(u(r),r);
> dsolve(ode1);
> u(r) := C1+C2/r;
> ode2:=r*diff(N(r),r)+N(r)-1+r^2*(diff(u(r),r))^2+1/r^2+Lambda*r^2;
> N(r) = -1/3*Lambda*r^2+1+1/r^2*C2^2+1/(r^2)+1/r*C3;
```

The expressions for $u(r)$ şi $N(r)$ contains three arbitrary constants of integrations C_1, C_2 and C_3. Choosing the adequate values $C_1 = 1, C_2 = \sqrt{q^2 + g^2 - 1}, C_3 = -2GM$, these solutions describe a *colored black hole* with the electrical charge q and magnetical charge g [6, 11].

B. Non-commutative gauge theory. We constructed a model of gauge theory having $U_*(2)$ as gauge group over a non-commutative torsionless [2,6]. The $U(2)$ gauge fields, denoted by $A = A_\mu^a(x) T_a dx^\mu$, $a = 0,1,2,3$ [2, 5], are chosen now under the form

$$A = uT_3 dt + w\left(T_2 d\theta - \sin\theta T_1 d\varphi\right) + \cos\theta T_3 d\varphi + \mathrm{v} T_0\, dt\,, \tag{17}$$

where u, v, w are functions depending only the radial variable *r*. The connection coefficients are determined only by one unknown function $A(r)$: $\Gamma_{10}^0 = -A'/A, \Gamma_{11}^1 = A'/A$. We suppose that the function $A(r)$ determines also the non-commutativity parameters:

$$\theta^{10} = -\theta^{10} = 1/A, \theta^{23} = -\theta^{32} = b = const. \tag{18}$$

We introduce first the components of the gauge fields $e_\mu^a(x)$ and $A_\mu^a(x)$ in our program. It contains a number of computing procedures which allow to obtain the torsion and curvature tensors $T_{\mu\nu}^\rho, R_{\lambda\rho\sigma}^\nu, \widetilde{R}_{\lambda\rho\sigma}^\nu$ of the non-commutative space-time, then the covariant derivatives $\nabla_\lambda T_{\mu\nu}^\rho, \nabla_\lambda R_{\mu\nu}^{\rho\sigma}, \widetilde{\nabla}_\lambda R_{\mu\nu}^{\rho\sigma}, \nabla_\lambda \widetilde{R}_{\mu\nu}^{\rho\sigma}, \widetilde{\nabla}_\lambda \widetilde{R}_{\mu\nu}^{\rho\sigma}$ and $\nabla_\lambda \theta^{\mu\nu}, \widetilde{\nabla}_\lambda \theta^{\mu\nu}$ of the torsion $T_{\mu\nu}^\rho$, curvatures $R_{\lambda\rho\sigma}^\nu, \widetilde{R}_{\lambda\rho\sigma}^\nu$ and the bi-vector $\theta^{\mu\nu}$ respectively, the quantum deformations of first order $A_\mu^{(1)}, F_{\mu\nu}^{(1)}, G^{\mu\nu(1)}$ for the gauge fields, the tensor associated to gauge fields and the metric of the space-time. The computing instructions for obtaining some of these quantities are:

```
> grdef(`T {^rho miu niu}:=Γ {^rho miu niu}-Γ {^rho niu miu}`);
> grdef(`R {^niu lambda rho sigma}:= Γ {^niu sigma lambda, rho}-Γ {^niu rho lambda,
  sigma}+Γ {^niu rho tau} * Γ {^tau sigma lambda}-Γ {^niu sigma tau} * Γ {^tau rho
  lambda}`);
> grdef(`R̃ {^niu lambda rho sigma}:= Γ {^niu lambda sigma, rho}-Γ {^niu lambda rho,
  sigma}+Γ {^niu tau rho } * Γ {^tau lambda sigma}-Γ {^niu tau sigma} * Γ {^tau lambda
  rho}`);
```

> grdef(`theta{^miu ^niu}`);

> grdef(` $\widetilde{R}$ {^miu ^niu rho sigma}:=theta{^miu ^lambda} $*$ $\widetilde{R}$ {^niu lambda rho sigma}`);

>grdef(` $D\theta$ {lambda ^miu ^niu}:= theta{^miu ^niu, lambda}+ $\Gamma$ {^miu sigma lambda} $*$ theta{^sigma ^niu}+ $\Gamma$ {^niu sigma lambda} $*$ theta{^miu ^sigma}`);

> grdef(` $F1$ {^a miu niu}:= -theta{^rho ^sigma} $*$ (A {^b rho} $*$ (DF {^c miu niu, sigma}+ Dg {^c miu niu sigma})-2 $*$ F {^b miu rho} $*$ F {^c niu sigma}) $*$ d {^a b c}/4;

Having these quantities, we write then the field equations and obtain the analytical/numerical solutions. The corresponding procedures and instructions are presented in section **2.2** (analytical solutions) and **2.3** (numerical solutions).

2.2. Computing of analytical solutions

As an example, we compute the solution of the field equation satisfied by the gauge potential $y(r) \equiv A(r)$ corresponding to the non-commutative $U_*(2)$ gauge theory [2, 8]. The instructions are:

> *ode*:= *diff*(*y*(*r*),*r*,*r*)* *y*(*r*) = 2* *diff*(*y*(*r*),*r*)^2;

> *dsolve*(*ode*, *y*(*r*));

and the program gives the solution $y(r) = -C_1/(r + C_2)$, where C_1 and C_2 are arbitrary constants of integration whose values are determined by the initial conditions: $y(0) = 1,\ y'(0) = 1$. Then, we obtain $C_1 = 1, C_2 = -1$ and the solution becomes $y(r) = 1/(1-r)$. The plot of this function (see section **2.3**) can be obtained by using the instructions:

> *y*(*r*):=1/(1-*r*); > *plot*(y(*r*), r = -2..2, y = -2..2);

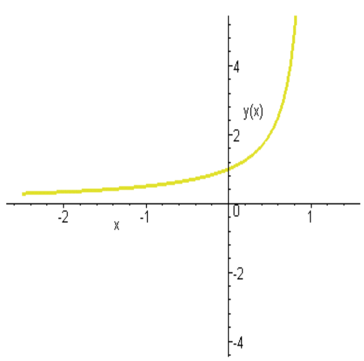
The program allows us to obtain also the analytical solutions for a system of coupled equations, for example the equations (15) and (16). If the considered system do not admit analytical solutions, then we must use numerical methods as it is described in the following section.

2.3. Obtaining numerical solutions and 2D and 3D plots

The computing program allows us to solve numerically some differential equations and plot the obtained results. We give the computing instructions also for the case of above equation:

```
with(DEtools):
> DEplot(diff(y(x),x$2)*y(x)=2*diff(y(x),x)^2,y(x),
> x = -2.5..1.4,[[y(0)=1,D(y)(0)=1]],y=-4..5,stepsize=.05);
de1 := {(D@@2)(x)(t)*x(t)=2*diff(x(t),t)^2}:
> init1 := {x(0)=1, D(x)(0)=1}:
> F := dsolve(de1 union init1, {x(t)},type=numeric, method=mgear,
> value=array([0,-.2,-.4,-.6,-.8,0,.2,.3,.4,.5,.6,.8]));
```

The table with numerical values and the **2D-**plot are presented below. From the table we can see that the program give also and numerical values for the derivative of first order of unknown function *A*(*r*).



r	$A(r)$	$A'(r)$
- 0.8	0.5555556658	0.3086420107
- 0.6	0.6250001475	0.3906251679
- 0.4	0.7142857491	0.5102039807
- 0.2	0.8333333391	0.6944443349
0	1	1
0.2	1.250000270	1.562501224
0.4	1.666667835	2.777784075
0.6	2.500004736	6.250033069
0.8	5.000030784	25.00036772

In the case when the solution depends of two variables, we can use the package *plot3d*, specifying the domain of values we need. For example, in order to obtain the **3D**-plot of the following function $\sin\theta/(1-r)$, we use the instructions:

> *plot3d*({*sin*(*theta*)/(1-*r*)}, *theta* = -*Pi*..*Pi*, *r* = -3..3); if $\theta\in(-\pi,\pi)$, $r\in(-3,3)$.

Objective 3. Computer-assisted study of the gauge field dynamics

3.1. Formulation of a dynamic model of gauge theory with $SU(n)\times SO(p,q)$ as structure group

The gauge potentials of the model are denoted by $A_\mu^\alpha(x)$, $\alpha=1,2,\ldots,n^2-1$ - in the case of the group $SU(n)$ and by $A_\mu^{ij}(x)=-A_\mu^{ji}(x)$, $e_\mu^i(x)$, $i,j=0,1,\ldots,m-1$ - in the case of the group $SO(p,q)$. They describe the internal non-abelian gauge fields and gravitational field respectively. If we use the general definition (2), then we can construct the tensors $F_{\mu\nu}^{ij}(x)$ and $G_{\mu\nu}^\alpha(x)$ associated to these fields. The integral of action for this system of fields has the expression

$$S_{EYM}=\int d^4x\,e\left[-\frac{1}{16\pi G}F-\frac{1}{4g^2}G_{\mu\nu}^\alpha G_\alpha^{\mu\nu}\right], \tag{19}$$

where $F=F_{\mu\nu}^{ij}e_i^\mu e_j^\nu$ and $e=\det\left(e_\mu^i\right)$. This integral corresponds to the standard theory of gravitation interacting with the non-abelian gauge fields. As an application we consider the case when the gauge group is $SU(2)\times SO(4,1)$ [see section **2.1]**.

We introduce into our model also a scalar matter field $\phi(t,x)$, having the mass *m*, which is described by the Lagrangian

$$L=\frac{1}{2}\left[\left(\frac{\partial\phi}{\partial t}\right)^2-\left(\frac{\partial\phi}{\partial x}\right)^2\right]+\frac{1}{2}m^2\phi^2-\frac{1}{4}f\phi^4, \tag{20}$$

where *f* is the constant of self-interaction. Thus, we obtain a dynamic model whose vacuum state is doubly degenerated: $\phi_v=\pm m/\sqrt{f}$ (this shows a spontaneous breaking of symmetry). The Lagrangian (20) gives the following field equation for $\phi(t,x)$:

$$\frac{\partial^2\phi}{\partial t^2}-\frac{\partial^2\phi}{\partial x^2}=m^2\phi-f\phi^3. \tag{21}$$

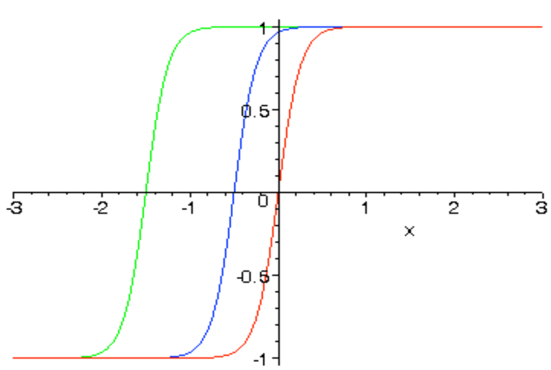
3.2. Construction of computing program

We use the routines *dsolve, PDEtools* and *plot3d* in order to determine the solution of the equation (21) and plot the function $\phi(t,x)$. For such a purpose, we use the following sequence of instructions:

```
> PDE := diff(phi(t,x),t,t)-diff(phi(t,x),x,x)=phi(t,x)-phi(t,x)^3;
> struc := pdsolve(PDE,HINT=f(t)*g(x));
> plot3d(phi(t,x), t=-1..1, x=-1..1);
```

The solution obtained with this program is

$$\phi(x,t)=\tanh\left(-C_1-C_2t+\frac{\sqrt{4C_2{}^2+2}\,x}{2}\right)$$



(22)

For any fixed values of the time *t,* it is a *solution of kink-type* (see the figure for the values $t = 0,2,6$ of the time). The integration constants C_1, C_2 have been convenient chosen for the three curves. Therefore, by this way we can study the dynamic evolution of the field $\phi(t,x)$ in the considered model of gauge theory with spontaneous symmetry breaking.

3.3. Simulation of interaction processes between matter fields mediated by gauge fields

The quantization of gauge fields and the study of different types of interactions is obtained by the method of *functional integral*. We consider a system containing a multiplet $\psi^a(x)$ of spinorial fields, a multiplet $\phi^a(x)$ of charged scalar fields, a multiplet $A_\mu^k(x)$ of gauge fields and a system of ghost fields $c^k(x)$. The Lagrangian for such a system contains terms corresponding to each multiplet of fields and also the interaction terms. For example, the gauge fields with *gauge* $-\alpha$ and the spinorial fields interacting with gauge fields are described, respectively, by the following terms:

$$L_1 = -\frac{1}{4}F_{\mu\nu}^k F_k^{\mu\nu} - \frac{1}{2\alpha}\left(\partial_\mu A^{k\mu}\right)^2,$$

$$L_2 = \overline{\psi}^a \gamma^\mu \left(\partial_\mu + igT_k A_\mu^k\right)_{ab}\psi^b - M\overline{\psi}^a\psi_a, \tag{23}$$

where g is the *coupling constant*. The Propagator of the gauge fields in momentum representation has the form

$$D_{\mu\nu}^{kl}(k) = \frac{\delta_{kl}}{k^2}\left[g_{\mu\nu} - (1-\alpha)\frac{k_\mu k_\nu}{k^2}\right]. \tag{24}$$

With these results we can compute the expressions associated to any vertex or loop of interaction from a Feynman diagram. For example, in the cases of spinorial and gauge field or the self-interaction of gauge fields we have, respectively, the following expressions: $g\gamma_\alpha (T_a)_{cb}$,

$$-ig^2 f_{lac} f_{lbd}\left(g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\beta\gamma}\right) - ig^2 f_{lad} f_{lbc}\left(g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\gamma}g_{\beta\delta}\right) - ig^2 f_{lab} f_{lcd}\left(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}\right). \tag{25}$$

In our works, we developed a computing program of the above expressions which allows to simulate different interactions between matter and/or gauge fields. We give here some instructions for computing the first vertex expression of (25) in the case of $SU(2)$ group:

```
> grdef(`gamma0{miu niu}`); grdef(`gamma1{miu niu}`); grdef(`gamma2{miu niu}`);
  grdef(`gamma3{miu niu}`);
> grdef(`T1{c b}`); grdef(`T2{c b}`); grdef(`T3{c b}`);
> grcalc(gamma0(dn,dn), gamma1(dn,dn), gamma2(dn,dn), gamma3(dn,dn), T1(dn,dn));
T2(dn,dn)); T3(dn,dn));
>grdef(`V02{a b c d}:= g*gamma0{a b}*T2{c d};
> grdisplay(_);
```

Analogous, we can compute the metric components $g_{\alpha\beta}$ and the constants of structure f_{abc} of $SU(2)$ group in order to obtain the expression of the second vertex in (25). For any other vertex or loop, the calculations are very laborious, and this shows us the utility of computing programs in obtaining of the corresponding results.

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